Backtracking Algorithms

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Backtracking Algorithms

- Systematically exhausted search the sample space, if any one get a solution, the algorithm stop. If we reach a point which is undesirable, undo the last step and try an alternative. This might force another undo, and so forth.
- Most backtracking algorithms are convenient to be implemented by recursion.
- A clever implement of exhaustive search by not try all possibilities.
- applications: 8 queens problem

Pruning

eliminate a large group of possibilities in 1 step

- postorder traversal to do $\alpha\beta$-pruning
  - α-pruning: the α value of a max position is defined to be the minimum possible value for that position. If the value of a min position is determined to be less than or equal to the α value of its parent, then we may stop generation of the remaining children of this min position.
  - β-pruning: the β value of a min position is defined to be the maximum possible value for that position. If the value of a max position is determined to be greater than or equal to the α value of its parent, then we may stop generation of the remaining children of this max position.
  - $\alpha\beta$-pruning: combine both α- and β-pruning. limits the searching to only $O(N^{1/2})$ nodes, where N is the size of the full game tree. Searches using $\alpha\beta$-pruning can go twice as deep as compared to an un-pruned tree.
- Exploiting symmetry: avoid to recomputed symmetric solution

Reconstruction Problem

- Given N points located on the x-axis, we can construct the bag of distances between all points D in $O(N^2)$ because $|D| = N(N - 1)/2$. Sort the distance: $O(N^2 \lg N)$
- The turnpike reconstruction problem: reconstruct the points from the distances bag.
- Backtracking to solve the reconstruction problem

Bandwidth Minimization

- Input a graph G, the goal is to find a permutation of the vertices on a line that minimize the maximum length of any edge.
- For a complete binary tree with 15 nodes, the pretty layout {10, 11, 5, 9, 8, 4, 2, 1, 3, 6, 12, 13, 7, 14, 15} has a longest edge of length 4, but a seemingly cramped layout {4, 8, 2, 9, 5, 1, 10, 11, 3, 6, 12, 7, 13, 14, 15} realizes the optimal bandwidth of 3.

Games

- A computer might use to play strategic game. The strategy requires the programmer, but not the computer, to do most of the thinking.
- Game tree: each node of the tree represents a board position, and the root is the starting position
- A successor position of P is any position P_s that is reachable from P by playing one legal move.
- Terminal position: leaves of the game tree
- Evaluation function: quantify the "goodness" of a position. A position that is a win for a computer gets the value +1; a draw gets a 0; and a lost gets a -1.
- Minimax strategy: if a position is not terminal, the value of the position is determined by recursively assuming optimal play by both sides. One player is trying to minimize the value of the position, while the other player is trying to maximize it.
- For more complex games, it is obviously infeasible to search all the way to the terminal nodes. In this case, we have to stop the search after a certain depth of recursion is reached.
- Ply: the number of moves of look-ahead, and it is equal to the depth of the recursion.
- To increase the look-ahead factor by evaluating fewer nodes without losing any information. Use a table to keep track of all position that have been evaluated, thus the second occurrence of a position need not be recomputed. The table is called transposition table and implemented by hashing.

Combine Backtracking With Other Algorithms

- backtracking with greedy method: the “show me the money” problem
- backtracking with randomized algorithm: the 8 queens problem