Project 1: Linear and Nonlinear Dynamic Systems

The purpose of this project is to highlight some of the most important differences between linear and nonlinear dynamic systems. In each problem, you will be asked to do some analytical work as well as some numerical simulation. You will also be required to interpret your results, and draw appropriate conclusions about the system equilibria and their stability properties.

Note: Before you begin working on this project, make sure you go over Demo 1.

Problem 1. Consider the first order *linear* differential equation

$$\dot{x} = -0.5x + p$$

in which p represents a variable parameter.

(a) Set $p = 0$, and compute the system equilibria. How many are there?

(b) Solve the equation numerically for initial conditions $x_0 = -2.5$, $x_0 = 0$ and $x_0 = 2.5$ (use $t = 0 : 0.01 : 20$). Plot all three solutions on a *single* diagram, and use this diagram to determine whether the system equilibria are stable.

Problem 2. In this problem, we will continue to work on the system described in Problem 1, this time setting the parameter value to $p = 2$.

(a) Compute the equilibria of the new system. How many are there?

(b) Solve the equation numerically for the same three initial conditions as in Problem 1 (use $t = 0 : 0.01 : 20$ once again). Plot all three solutions on a *single* diagram, and use this diagram to determine whether the new equilibria have the same stability properties as the ones for $p = 0$. Explain your conclusions. Note: For a better view of the solutions, set the data bounds for the y-axis to $[-3 \;\; 5]$.

Problem 3. Consider the linear differential equation

$$\dot{x} = 0.5x + p$$

where p is once again a parameter. This system is very similar to the one in Problems 1 and 2, except for the sign of the term in front of variable x.

(a) Set $p = 0$, and compute the system equilibria. Are they any different than in Problem 1?

(b) Solve the equation numerically for initial conditions $x_0 = 0.01$, $x_0 = 0$ and $x_0 = -0.01$ (using $t = 0 : 0.01 : 10$). Plot all three solutions on a *single* diagram, and use this diagram to determine whether the system equilibria are stable. Why do you think this system is so sensitive to the choice of initial conditions? Explain.

Problem 4. In this problem, we will examine a simple *nonlinear* system of the form

$$\dot{x} = \sin x + p$$

(a) Set $p = 0$, and compute the system equilibria. How many are there? How is your answer different from the the answer you obtained in Problems 1 - 3? Explain.

(b) Solve the equation numerically for the following initial conditions: $x_0 = -10$; $x_0 = -8$; $x_0 = -6$; $x_0 = -4$; $x_0 = -2$; $x_0 = 2$; $x_0 = 4$; $x_0 = 6$; $x_0 = 8$; $x_0 = 10$ (use $t = 0 : 0.01 : 10$ in all cases). Plot all 10 solutions on a *single* diagram, and use this diagram to determine the stability of the system equilibria that appear in it. Are all the equilibria that you computed represented in this graph? Explain.

(c) Solve the equation numerically for initial conditions $x_0 = 0.01$, $x_0 = 0$ and $x_0 = -0.01$ (using $t = 0 : 0.01 : 10$). Which equilibria appear in this plot, and what can you say about their stability properties? Do you notice any similarities with the plot you obtained in Problem 3b? Are there any significant differences? Explain.

Problem 5. Take another look at the system given in Problem 4, this time with $p = 2$. Solve the equation numerically for initial conditions $x_0 = -5$, $x_0 = 0$ and $x_0 = 5$ (using $t = 0 : 0.01 : 10$), and plot all three solutions on a *single* diagram. What has changed with respect to the case when $p = 0$? Why do you think the system behavior for $p = 0$ and $p = 2$ is so different? Explain.

Problem 6. In this problem, we will examine a *nonlinear* system of the form

$$\dot{x} = x(p - x^2)$$

(a) Set $p = -2$, and compute the system equilibria. How many are there?

(b) Solve the equation numerically for initial conditions $x_0 = -2$, $x_0 = 0$ and $x_0 = 2$ (with $t = 0 : 0.01 : 5$), and plot all three solutions on a *single* diagram. Use this diagram to determine the stability properties of the system equilibria.

Problem 7. Consider the same system as in Problem 6, this time with $p = 2$.

(a) How many equilibria does the system have in this case?

(b) Solve the equation numerically for the following set of initial conditions: $x_0 = -2$; $x_0 = -1$; $x_0 = -0.01$; $x_0 = 0$; $x_0 = 0.01$; $x_0 = 1$ and $x_0 = 2$ (use $t = 0 : 0.01 : 5$ in all cases). Plot all 7 solutions on a *single* diagram, and use this diagram to determine the stability properties of the system equilibria. How does the behavior of this system differ from the case when $p = -2$? Explain.