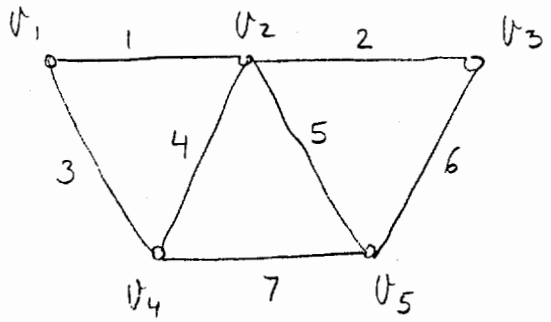
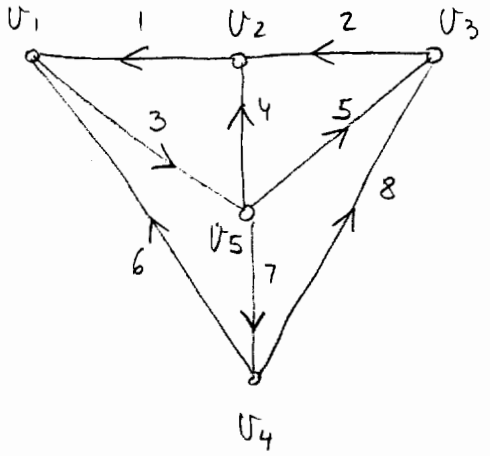
HOMEWORK #3

① For the graph shown below, select tree $T=\{1,3,6,7\}$ and identify the fundamental cutsets and loops.



② For the directed graph shown below



a) Write the KCL and KVL equations for the meshes and nodes with v_5 as the reference node

b) Chose tree $T=\{3,4,5,7\}$ and repeat part a) using cutsets for KCL

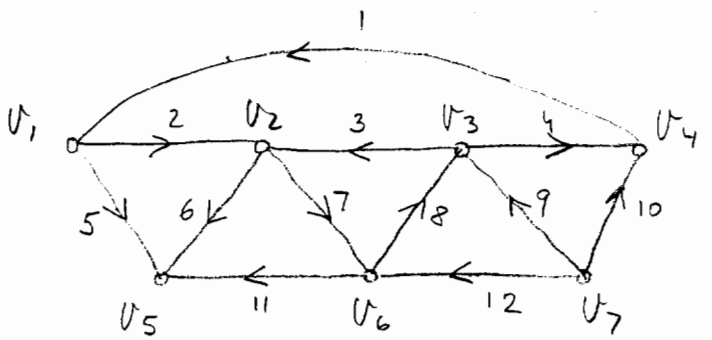
c) Compare the equations in parts a) & b).

③ Find the directed graph that corresponds to each of the following incidence matrices

a) $$A_1 = \begin{bmatrix} -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & -1 & -1 \end{bmatrix}$$

b) $$A_2 = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

④ For the connected graph below



select tree $T = \{2, 3, 4, 6, 7, 10\}$ and find matrices.

(a) B_f

(b) Q_f

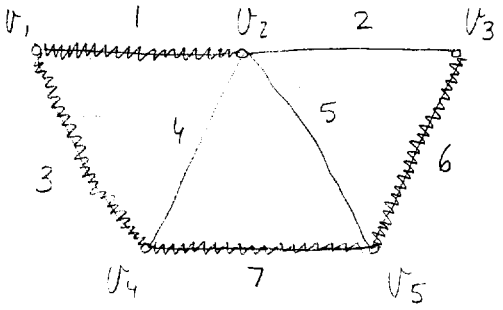
(c) A

⑤ Find a graph whose fundamental loop matrix Bf has the form:

$$B_f = \begin{array}{c} \begin{matrix} e_1 & e_4 & e_7 & e_8 & e_2 & e_3 & e_5 & e_6 \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 1 \end{bmatrix} \end{array}$$

SOLUTIONS TO HMWK #3

(1)



$T = \{1, 3, 6, 7\}$

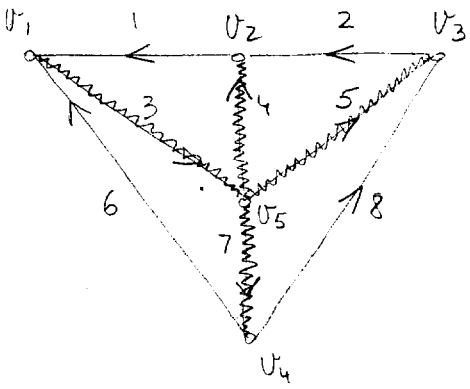
C_1: $\{\underline{1}, 2, 4, 5\}$ $\quad$ C_3: $\{\underline{6}, 2\}$

C_2: $\{\underline{3}, 2, 1, 6\}$ $\quad$ C_4: $\{\underline{7}, 2, 5\}$

L_1: $\{\underline{2}, 6, 7, 3, 1\}$ $\quad$ L_3: $\{\underline{5}, 7, 3, 1\}$

L_2: $\{\underline{4}, 3, 1\}$

(2)



a) KVL (meshes)

1) $V_1 + V_3 + V_4 = 0$

2) $V_2 - V_4 + V_5 = 0$

3) $V_6 + V_3 + V_7 = 0$

4) $V_8 - V_5 + V_7 = 0$

KCL (nodes)

1) $I_3 - I_1 - I_6 = 0$

2) $-I_4 + I_1 - I_2 = 0$

3) $-I_5 + I_2 - I_8 = 0$

4) $-I_7 + I_6 + I_8 = 0$

b) KVL (FUNDAMENTAL LOOPS)

Exactly the same as in part a)

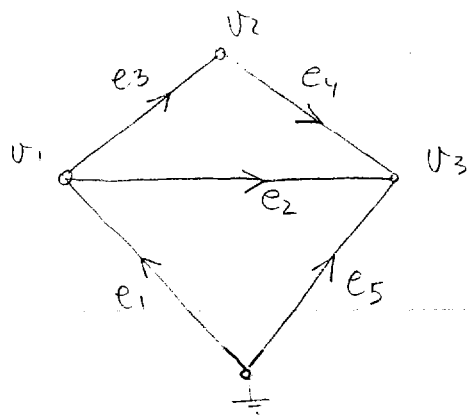
KCL (FUNDAMENTAL CUTSETS)

1) $I_3 - I_1 - I_6 = 0$
2) $I_4 + I_2 - I_1 = 0$
3) $I_5 + I_8 - I_2 = 0$
4) $I_7 - I_6 - I_8 = 0$

c) The equations in a) and b) are identical. It should be pointed out that this is the case for that particular choice of tree. For some other tree, this will not hold.

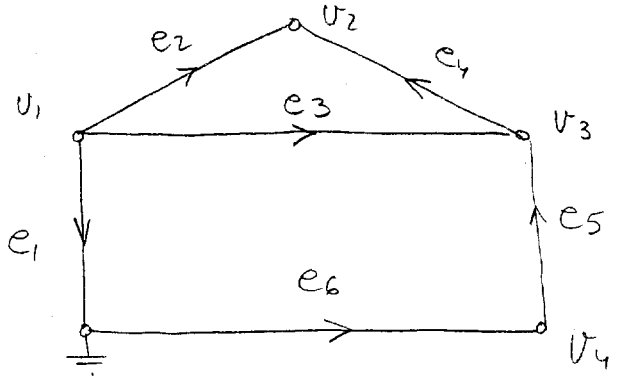
(3) a)

$$A_1 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \end{array} \begin{array}{c} \begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_5 \end{array} \\ \begin{bmatrix} -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & -1 & -1 \end{bmatrix} \end{array}$$



b)

$$A_2 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \end{array}$$



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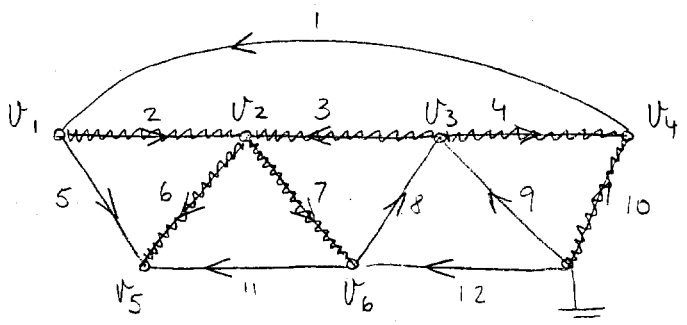

a)

$$B_f = \left[\begin{array}{cccccc|cccccc} e_1 & e_5 & e_8 & e_9 & e_{11} & e_{12} & e_2 & e_3 & e_4 & e_6 & e_7 & e_{10} \\ \hline 1 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 1 & 0 & -1 & -1 \end{array}\right]$$

b)

$$Q_f = \left[\begin{array}{cccccc|cccccc} e_1 & e_5 & e_8 & e_9 & e_{11} & e_{12} & e_2 & e_3 & e_4 & e_6 & e_7 & e_{10} \\ \hline -1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

c)

$$A = \left[\begin{array}{c|cccccc|cccccc} & e_1 & e_5 & e_8 & e_9 & e_{11} & e_{12} & e_2 & e_3 & e_4 & e_6 & e_7 & e_{10} \\ \hline v_1 & -1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ v_2 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 1 & 1 & 0 \\ v_3 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ v_4 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 \\ v_5 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ v_6 & 0 & 0 & 1 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & -1 & 0 \end{array}\right]$$

(5)