

## HOMEWORK # 2

1. For the matrix below

$$A = \begin{bmatrix} 1 & 5 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 3 \end{bmatrix}$$

use Gaussian elimination to:

- a) Compute  $\det(A)$ .
- b) Compute the inverse of  $A$ .
- c) Solve the system

$$\begin{bmatrix} 1 & 5 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

2. Use Gaussian elimination to determine the rank of matrix

$$A = \begin{bmatrix} 16 & 16 & -16 & -32 \\ 12 & 12 & 8 & 16 \\ -30 & -30 & -20 & -40 \\ 56 & 56 & 84 & 168 \end{bmatrix}$$

## SOLUTIONS TO HWK #2

1. a)  $\det(A) = -22$

b)

$$A^{-1} = \frac{1}{-22} \begin{bmatrix} -2 & 14 & -4 \\ 5 & -2 & -1 \\ -1 & -4 & 9 \end{bmatrix} = \begin{bmatrix} -0.0909 & 0.6364 & -0.1818 \\ 0.2273 & -0.0909 & -0.0455 \\ -0.0455 & -0.1818 & 0.4091 \end{bmatrix}$$

c)

$$x = \frac{1}{-22} \begin{bmatrix} 4 \\ 1 \\ 13 \end{bmatrix} = \begin{bmatrix} 0.1818 \\ 0.0455 \\ 0.5909 \end{bmatrix}$$

2. The first three steps of Gaussian elimination yield:

$$\begin{bmatrix} 16 & 16 & -16 & -32 \\ 0 & 0 & 20 & 40 \\ 0 & 0 & -50 & -100 \\ 0 & 0 & 140 & 280 \end{bmatrix}$$

After the next two steps, we obtain:

(2)

$$\begin{bmatrix} 16 & 16 & -16 & -32 \\ 0 & 0 & 20 & 40 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rearranging columns two and three

$$\left[ \begin{array}{cc|cc} 16 & -16 & 16 & -32 \\ 0 & 20 & 0 & 40 \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

we identify a non-singular  $2 \times 2$  submatrix.  
Consequently,  $\text{rank}(A) = 2$ .