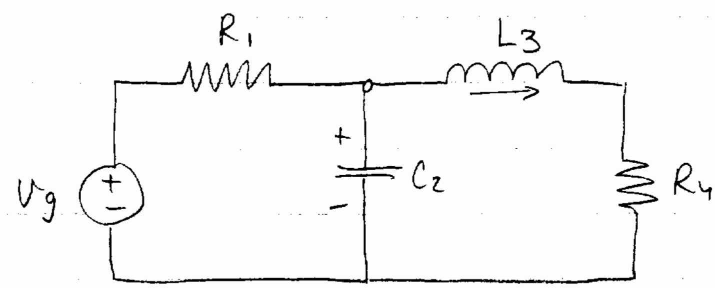
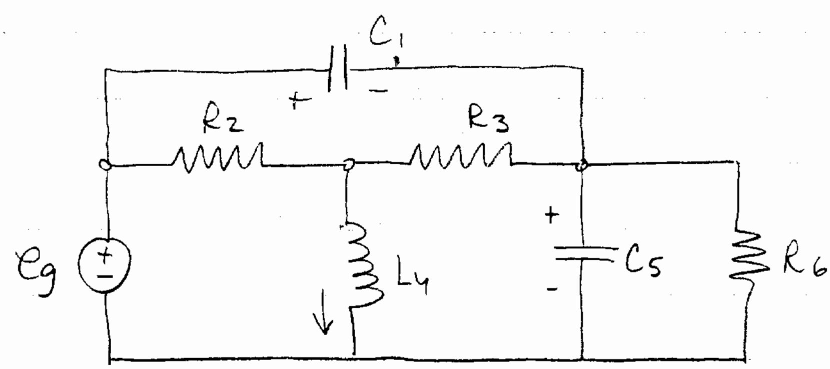
HOMEWORK # 5

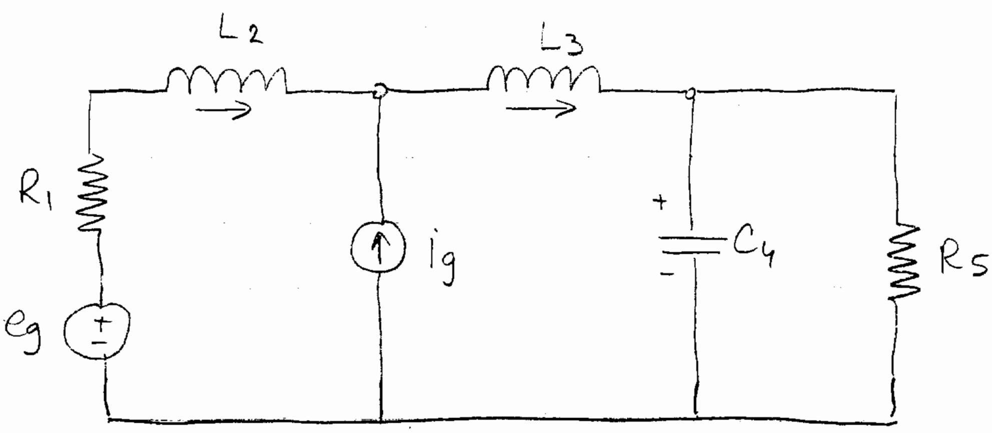
(1) Obtain the state equations for the circuit below:



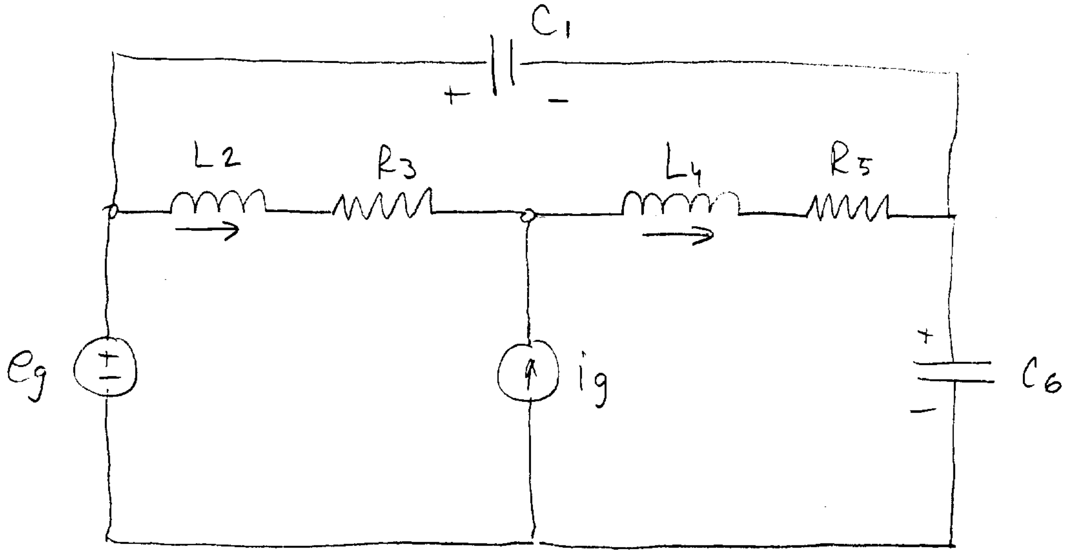
(2) Repeat Problem 1 for the following circuit:



(3) Formulate the state equations for the circuit below:

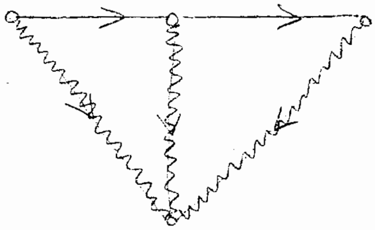


(4) Determine the state equations for the following circuit



SOLUTIONS TO HWK #5

(1)



1) $i_{C2} + i_{L3} - i_{R1} = 0 \implies C_2 \frac{dV_{C2}}{dt} = i_{R1} - i_{L3}$

2) $V_{L3} + V_{R4} - V_{C2} = 0 \implies L_3 \frac{di_{L3}}{dt} = V_{C2} - V_{R4}$

To get rid of i_{R1}:

$V_{R1} + V_{C2} - E_g = 0 \implies i_{R1} = \frac{E_g - V_{C2}}{R_1}$

To get rid of V_{R4}:

$i_{R4} - i_{L3} = 0 \implies V_{R4} = R_4 i_{L3}$

Therefore:

$$C_2 \frac{dV_{C2}}{dt} = -\frac{1}{R_1} V_{C2} - i_{L3} + \frac{1}{R_1} E_g$$

$$L_3 \frac{di_{L3}}{dt} = V_{C2} - R_4 i_{L3}$$

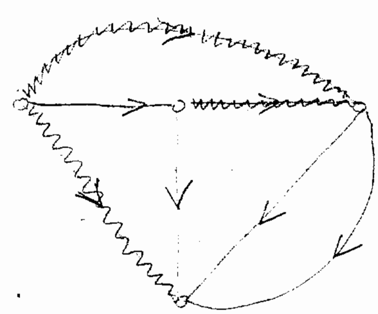
in matrix form:

$$\begin{bmatrix} \frac{dV_{C2}}{dt} \\ \frac{di_{L3}}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{1}{R_1C_2} & -\frac{1}{C_2} \\ \frac{1}{L_3} & -\frac{R_4}{L_3} \end{bmatrix} \begin{bmatrix} V_{C2} \\ i_{L3} \end{bmatrix} + \begin{bmatrix} \frac{1}{R_1C_2} \\ 0 \end{bmatrix} E_g$$

(2) This circuit has one degenerate loop which implies:

$$V_{C5} = E_g - V_{C1}$$

Consequently, V_{C5} will not be a state variable only V_{C1} and i_{L4} will.



1) $i_{C1} + i_{R2} - i_{L4} - i_{C5} - i_{R6} = 0 \Rightarrow C_1 \frac{dV_{C1}}{dt} = i_{L4} + i_{C5} - i_{R2} + i_{R6}$

2) $V_{L4} - E_g + V_{C1} - V_{R3} = 0 \Longrightarrow L_4 \frac{di_{L4}}{dt} = -V_{C1} + V_{R3} + E_g$

To get rid of i_{R2} and V_{R3}:

$$V_{R2} + V_{R3} - V_{C1} = 0 \Longrightarrow R_2 i_{R2} + V_{R3} = V_{C1}$$

$$i_{R3} + i_{L4} - i_{R2} = 0 \Longrightarrow -i_{R2} + \frac{1}{R_3} V_{R3} = -i_{L4}$$

We can rewrite this as:

$$\begin{bmatrix} R_2 & 1 \\ -1 & \frac{1}{R_3} \end{bmatrix} \begin{bmatrix} i_{R2} \\ V_{R3} \end{bmatrix} = \begin{bmatrix} V_{C1} \\ -i_{L4} \end{bmatrix}$$

and solve for i_{R2} and V_{R3}. The determinant is

$$\Delta = \frac{R_2}{R_3} + 1 = \frac{R_2 + R_3}{R_3}$$

Therefore,

$$\begin{bmatrix} i_{R2} \\ V_{R3} \end{bmatrix} = \frac{R_3}{R_2 + R_3} \begin{bmatrix} \frac{1}{R_3} & -1 \\ 1 & R_2 \end{bmatrix} \begin{bmatrix} V_{C1} \\ -i_{L4} \end{bmatrix}$$

Explicitly, we get:

$$\boxed{\begin{aligned} i_{R2} &= \frac{1}{R_2 + R_3} V_{C1} + \frac{R_3}{R_2 + R_3} i_{L4} \\ V_{R3} &= \frac{R_3}{R_2 + R_3} V_{C1} - \frac{R_2 R_3}{R_2 + R_3} i_{L4} \end{aligned}}$$

To get rid of i_{R6}:

$$V_{R6} - E_g + V_{C1} = 0 \implies \boxed{i_{R6} = \frac{E_g - V_{C1}}{R_6}}$$

To get rid of i_{C5}:

$$i_{C5} = C_5 \frac{dV_{C5}}{dt} = C_5 \frac{d}{dt}(E_g - V_{C1}) = C_5 \frac{dE_g}{dt} - C_5 \frac{dV_{C1}}{dt}$$

We can now rewrite the equations as:

$$C_1 \frac{dV_{C1}}{dt} = i_{L4} + C_5 \frac{dE_g}{dt} - C_5 \frac{dV_{C1}}{dt} - \frac{1}{R_2+R_3} V_{C1} - \frac{R_3}{R_2+R_3} i_{L4} + \frac{E_g - V_{C1}}{R_6}$$

$$L_4 \frac{di_{L4}}{dt} = -V_{C1} + \frac{R_3}{R_2+R_3} V_{C1} - \frac{R_2 R_3}{R_2+R_3} i_{L4} + E_g$$

Grouping the terms:

$$(C_1+C_5) \frac{dV_{C1}}{dt} = -\left(\frac{1}{R_6} + \frac{1}{R_2+R_3} \right) V_{C1} + \frac{R_2}{R_2+R_3} i_{L4} + \frac{1}{R_6} E_g + C_5 \frac{dE_g}{dt}$$

$$L_4 \frac{di_{L4}}{dt} = -\frac{R_2}{R_2+R_3} V_{C1} - \frac{R_2 R_3}{R_2+R_3} i_{L4} + E_g$$

In matrix form:

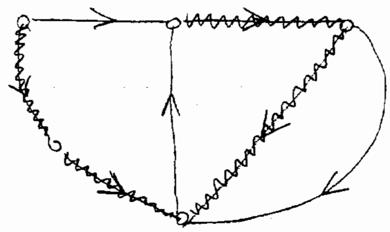
$$\begin{bmatrix} \frac{dV_{C1}}{dt} \\ \frac{di_{L4}}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{(R_2+R_3+R_6)}{R_6(R_2+R_3)(C_1+C_5)} & \frac{R_2}{(R_2+R_3)(C_1+C_5)} \\ \frac{-R_2}{L_4(R_2+R_3)} & -\frac{R_2 R_3}{L_4(R_2+R_3)} \end{bmatrix} \begin{bmatrix} V_{C1} \\ i_{L4} \end{bmatrix} + \begin{bmatrix} \frac{1}{R_6(C_1+C_5)} \\ \frac{1}{L_4} \end{bmatrix} E_g$$

$$+ \begin{bmatrix} \frac{C_5}{C_1+C_5} \\ 0 \end{bmatrix} \frac{dE_g}{dt}$$

(3) This circuit has one degenerate cutset, which implies

$$i_{L3} = i_{L2} + i_g(t)$$

As a result, i_{L3} will not be a state variable - only i_{L2} and V_{C4} will.



1) $V_{L2} + V_{L3} + V_{C4} - E_g - V_{R1} = 0 \implies L_2 \dfrac{di_{L2}}{dt} = -V_{C4} - V_{L3} + V_{R1} + E_g$

2) $i_{C4} - i_{L2} - i_g + i_{R5} = 0 \implies C_4 \dfrac{dV_{C4}}{dt} = i_{L2} - i_{R5} + i_g$

To get rid of V_{R1}:

$i_{R1} + i_{L2} = 0 \implies V_{R1} = -R_1 i_{L2}$

To get rid of i_{R5}:

$V_{R5} - V_{C4} = 0 \implies i_{R5} = \dfrac{V_{C4}}{R5}$

To get rid of V_{L3}:

$$V_{L3} = L_3 \frac{di_{L3}}{dt} = L_3 \frac{d}{dt}\left(i_{L2} + i_g(t)\right) = L_3 \frac{di_{L2}}{dt} + L_3 \frac{di_g}{dt}$$

We now have:

$$L_2 \frac{di_{L2}}{dt} = -V_{C4} - L_3 \frac{di_{L2}}{dt} - L_3 \frac{di_g}{dt} - R_1 i_{L2} + E_g$$

$$C_4 \frac{dV_{C4}}{dt} = i_{L2} - \frac{V_{C4}}{R_5} + i_g$$

Grouping the terms:

$$(L_2+L_3) \frac{di_{L2}}{dt} = -R_1 i_{L2} - V_{C4} + E_g - L_3 \frac{di_g}{dt}$$

$$C_4 \frac{dV_{C4}}{dt} = i_{L2} - \frac{V_{C4}}{R_5} + i_g$$

In matrix form:

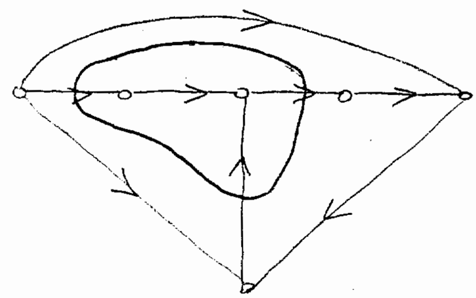
$$\begin{bmatrix} \frac{di_{L2}}{dt} \\ \frac{dV_{C4}}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R_1}{L_2+L_3} & -\frac{1}{L_2+L_3} \\ \frac{1}{C_4} & -\frac{1}{C_4 R_5} \end{bmatrix} \begin{bmatrix} i_{L2} \\ V_{C4} \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{L_2+L_3} \\ \frac{1}{C_4} & 0 \end{bmatrix} \begin{bmatrix} i_g \\ E_g \end{bmatrix} +$$

$$+ \begin{bmatrix} -\frac{L_3}{L_2+L_3} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{di_g}{dt} \\ \frac{dE_g}{dt} \end{bmatrix}$$

(4) In this circuit there is one degenerate loop and one degenerate cutset. The loop is obvious, implying:

$$V_{C6} = E_g - V_{C1}$$

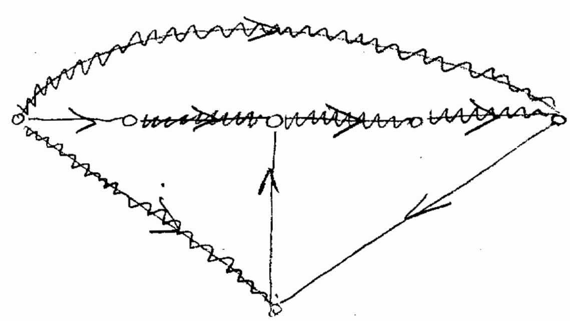
The degenerate cutset can be seen from the graph



This cutset implies

$$i_{L4} = i_{L2} + i_g$$

Since we have double degeneracy, the only state variables will be V_{C1} and i_{L2}.



1) $i_{C1} + i_{L2} + i_g - i_{C6} = 0 \quad \Rightarrow \quad C_1 \dfrac{dV_{C1}}{dt} = -i_{L2} + i_{C6} - i_g$

2) $V_{L2} + V_{R3} + V_{L4} + V_{R5} - V_{C1} = 0 \Rightarrow L_2 \dfrac{di_{L2}}{dt} = V_{C1} - V_{R3} - V_{R5} - V_{L4}$

To get rid of V_{R3}:

$i_{R3} = i_{L2} \quad \Rightarrow \quad V_{R3} = R_3 i_{L2}$

To get rid of V_{R5}:

$$i_{R5} - i_{L2} - i_g = 0 \quad \Rightarrow \quad V_{R5} = R_5(i_{L2} + i_g)$$

To get rid of i_{C6}:

$$i_{C6} = C_6 \frac{d}{dt}(E_g - V_{C1})$$

To get rid of V_{L4}:

$$V_{L4} = L_4 \frac{d}{dt}(i_{L2} + i_g)$$

We now have:

$$L_2 \frac{di_{L2}}{dt} = V_{C1} - R_3 i_{L2} - R_5 i_{L2} - R_5 i_g - L_4 \frac{di_{L2}}{dt} - L_4 \frac{di_g}{dt}$$

$$C_1 \frac{dV_{C1}}{dt} = -i_{L2} + C_6 \frac{dE_g}{dt} - C_6 \frac{dV_{C1}}{dt} - i_g$$

Grouping the terms:

$$(L_2 + L_4)\frac{di_{L2}}{dt} = -(R_3 + R_5) i_{L2} + V_{C1} - R_5 i_g - L_4 \frac{di_g}{dt}$$

$$(C_1 + C_6)\frac{dV_{C1}}{dt} = -i_{L2} - i_g + C_6 \frac{dE_g}{dt}$$

In matrix form, we can rewrite this as:

$$\begin{bmatrix} \dfrac{di_{L2}}{dt} \\[2ex] \dfrac{dV_{C1}}{dt} \end{bmatrix} = \begin{bmatrix} -\dfrac{(R_3+R_5)}{L_2+L_4} & \dfrac{1}{L_2+L_4} \\[2ex] -\dfrac{1}{C_1+C_6} & 0 \end{bmatrix} \begin{bmatrix} i_{L2} \\[2ex] V_{C1} \end{bmatrix} + \begin{bmatrix} -\dfrac{R_5}{L_3+L_4} & 0 \\[2ex] -\dfrac{1}{C_1+C_6} & 0 \end{bmatrix} \begin{bmatrix} I_g \\[2ex] E_g \end{bmatrix} +$$

$$+ \begin{bmatrix} -\dfrac{L_4}{L_2+L_4} & 0 \\[2ex] 0 & \dfrac{C_6}{C_1+C_6} \end{bmatrix} \begin{bmatrix} \dfrac{di_g}{dt} \\[2ex] \dfrac{dE_g}{dt} \end{bmatrix}$$