ELEN 211
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HOMEWORK # 1

1. Find the inverse Laplace transform of:

a)

$$\frac{s+3}{s^2+7s+10}$$

b)

$$\frac{2s+4}{s(s+1)(4s+2)}$$

c)

$$\frac{1}{(s-5)^2(s+7)}$$

d)

$$\frac{1}{s(s+2)^3}$$

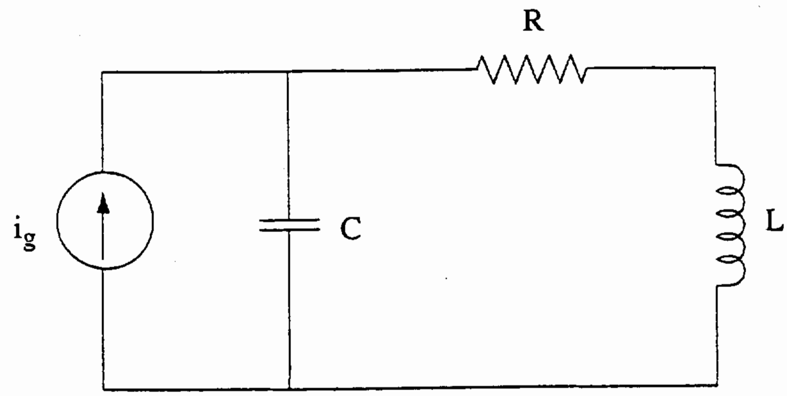
e)

$$\frac{2s+1}{3s(s^2+4s+13)}$$

f)

$$\frac{1}{(s^2+4)(s^2+9)}$$

2. In the circuit below



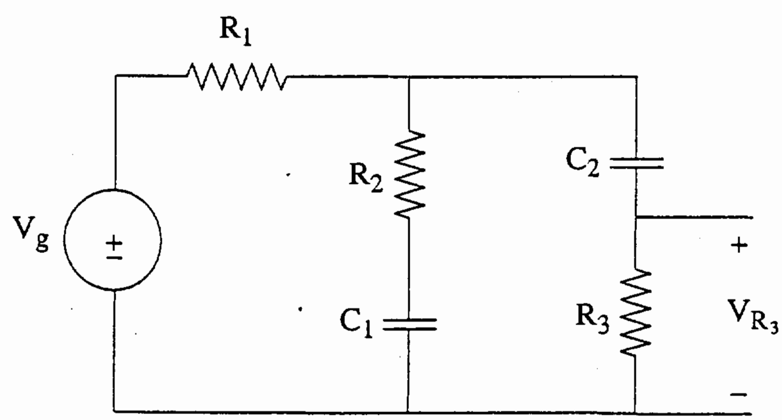
you are given

$R = 2\Omega$; $C = 0.5F$; $L = 1H$

a) Find an expression for $I_L(s)$ in terms of $I_g(s)$ and initial conditions $i_L(0)$ and $v_C(0)$.

b) Compute $i_L(t)$ in case $i_g(t)$ is a *unit step function*, $i_L(0) = 2$ and $v_C(0) = 1$.

3. In the circuit below



you are given

$R_1 = R_2 = R_3 = 1\Omega$; $C_1 = C_2 = 1/6$ F

and $v_g(t) = 3\cos t$. The initial conditions are zero.

a) Find an expression for $V_{R3}(s)$ in terms of $V_g(s)$.

b) Compute voltage $v_{R3}(t)$.

SOLUTIONS TO HMWK #1

1. a) $$\frac{s+3}{s^2+7s+10} = \frac{1/3}{s+2} + \frac{2/3}{s+5} \Rightarrow \boxed{\frac{1}{3}e^{-2t} + \frac{2}{3}e^{-5t}}$$

b) $$\frac{2s+4}{s(s+1)(4s+2)} = \frac{2}{4}\cdot\frac{(s+2)}{s(s+1)(s+1/2)} = \frac{1}{2}\left[\frac{4}{s} + \frac{2}{s+1} - \frac{6}{s+1/2}\right] =$$

$$= \frac{2}{s} + \frac{1}{s+1} - \frac{3}{s+1/2} \Rightarrow \boxed{2 + e^{-t} - 3e^{-1/2t}}$$

c) $$\frac{1}{(s-5)^2(s+7)} = \frac{-1/144}{s-5} + \frac{1/12}{(s-5)^2} + \frac{1/144}{s+7}$$

$$\Rightarrow \boxed{-\frac{1}{144}e^{5t} + \frac{1}{12}te^{5t} + \frac{1}{144}e^{-7t}}$$

d) $$\frac{1}{s(s+2)^3} = \frac{1/8}{s} - \frac{1/8}{s+2} - \frac{1/4}{(s+2)^2} - \frac{1/2}{(s+2)^3}$$

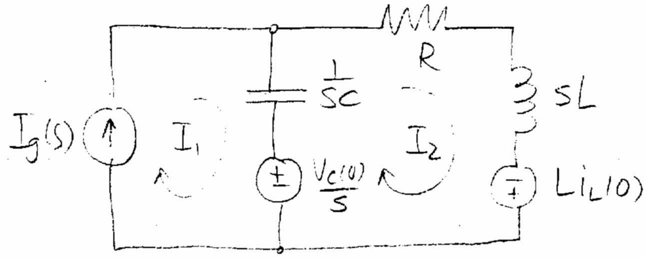
$$\Rightarrow \boxed{\frac{1}{8} - \frac{1}{8}e^{-2t} - \frac{1}{4}te^{-2t} - \frac{1}{4}t^2e^{-2t}}$$

e) $$\frac{2s+1}{3s(s^2+4s+13)} = \frac{1}{39}\left[\frac{1}{s} - \frac{(s+2)}{(s+2)^2+3^2} + 8\cdot\frac{3}{(s+2)^2+3^2}\right]$$

$$\Rightarrow \boxed{\frac{1}{39}\left[1 - e^{-2t}(\cos 3t - 8\sin 3t)\right]}$$

f) $$\frac{1}{(s^2+4)(s^2+9)} = \frac{1/5}{s^2+4} - \frac{1/5}{s^2+9} \Rightarrow \boxed{\frac{1}{10}\sin 2t - \frac{1}{15}\sin 3t}$$

2) The transformed circuit is



Note that $I_1(s) \equiv I_g(s)$, and is therefore already known. Consequently, we need only one KVL equation:

$$R I_2(s) + sL I_2(s) - L i_L(0) - \frac{V_C(0)}{s} + \frac{1}{sC}\left(I_2(s) - I_1(s)\right) = 0$$

Grouping the terms and replacing $I_1(s)$ by $I_g(s)$:

$$I_2(s)\left[sL + R + \frac{1}{sC}\right] = \frac{1}{sC} I_g(s) + L i_L(0) + \frac{V_C(0)}{s}$$

After multiplying both sides by s and substituting the numerical values for R, L and C:

$$I_2(s) = \frac{2}{s^2+2s+2} I_g(s) + \frac{s i_L(0) + V_C(0)}{s^2+2s+2}$$

b) Since $I_g(s) = \frac{1}{s}$, $i_L(0) = 2$ and $V_C(0) = 1$ we have

$$I_2(s) = \frac{2}{s(s^2+2s+2)} + \frac{2s+1}{s^2+2s+2}$$

Partial fraction expansion allows us to rewrite the first term as:

$$\frac{2}{s(s^2+2s+2)} = \frac{1}{s} - \frac{(s+2)}{s^2+2s+2}$$

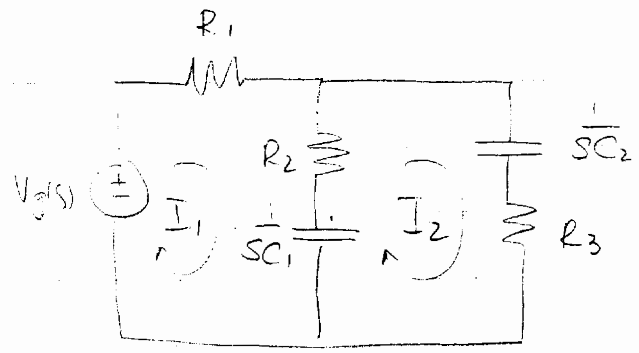
Consequently,

$$I_L(s) = \frac{1}{s} + \frac{s-1}{s^2+2s+2} = \frac{1}{s} + \frac{s+1}{(s+1)^2+1} - 2\cdot\frac{1}{(s+1)^2+1}$$

and

$$\boxed{i_L(t) = u(t) + e^{-t}\cos t - 2e^{-t}\sin t}$$

3. Since the initial conditions are zero, the transformed circuit becomes:



The two KVL equations for the meshes are:

$$\left(R_1+R_2+\frac{1}{sC_1}\right)I_1(s) - \left(R_2+\frac{1}{sC_1}\right)I_2(s) = V_g(s)$$

$$-\left(R_2+\frac{1}{sC_1}\right)I_1(s) + \left(R_2+R_3+\frac{1}{sC_1}+\frac{1}{sC_2}\right)I_2(s) = 0$$

Substituting element values and rewriting the equations in matrix form yields:

$$\begin{bmatrix} (2+\frac{6}{s}) & -(1+\frac{6}{s}) \\ -(1+\frac{6}{s}) & (2+\frac{12}{s}) \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} V_g(s) \\ 0 \end{bmatrix}$$

The determinant is

$$\Delta(s) = 3 + \frac{24}{s} + \frac{36}{s^2}$$

Thus,

$$\begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \frac{1}{\Delta(s)} \begin{bmatrix} (2+\frac{12}{s}) & (1+\frac{6}{s}) \\ (1+\frac{6}{s}) & (2+\frac{6}{s}) \end{bmatrix} \begin{bmatrix} V_g(s) \\ 0 \end{bmatrix}$$

and

$$I_2(s) = \frac{1+\frac{6}{s}}{3+\frac{24}{s}+\frac{36}{s^2}} \cdot V_g(s)$$

Multiplying the numerator and denominator by s^2 we obtain

$$\boxed{I_2(s) = \frac{s(s+6)}{3s^2+24s+36} V_g(s) = \frac{s(s+6)}{3(s+2)(s+6)} V_g(s) = \frac{s}{3(s+2)} V_g(s)}$$

Note also that $V_{R_3}(s) = 1 \cdot I_2(s)$, since $R_3 = 1\Omega$.

Since

$$V_g(s) = \frac{3s}{s^2+1}$$

we have

$$V_{R_3}(s) = \frac{s^2}{(s+2)(s^2+1)} = \frac{4/5}{s+2} + \frac{1/5\, s - 2/5}{s^2+1}$$

We can rewrite this as

$$V_{R_3}(s) = \frac{1}{5}\left[\frac{4}{s+2} + \frac{s}{s^2+1} - 2 \cdot \frac{1}{s^2+1}\right]$$

As a result

$$V_{R_3}(t) = \frac{1}{5}\left[4e^{-2t} + \cos t - 2\sin t\right]$$

Observing that

$$\frac{1}{5}(\cos t - 2\sin t) = \frac{1}{\sqrt{5}}\cos(t+63^\circ)$$

we can rewrite the answer as

$$\boxed{V_{R_3}(t) = \frac{4}{5}e^{-2t} + \frac{1}{\sqrt{5}}\cos(t+63^\circ)}$$