Backtracking Algorithms

Ming-Hwa Wang, Ph.D.
COEN 279/AMTH 377 Design and Analysis of Algorithms
Department of Computer Engineering
Santa Clara University

**Backtracking Algorithms**
- Systematically exhausted search the sample space, if any one get a
  solution, the algorithm stop. If we reach a point which is undesirable,
  undo the last step and try an alternative. This might force another undo,
  and so forth.
- Most backtracking algorithms are convenient to be implemented by
  recursion.
- A clever implement of exhaustive search by not try all possibilities.
  applications: 8 queens problem

**Pruning**
eliminate a large group of possibilities in 1 step
- postorder traversal to do αβ-pruning
  - α-pruning: the α value of a max position is defined to be the
    minimum possible value for that position. If the value of a min
    position is determined to be less than or equal to the α value of
    its parent, then we may stop generation of the remaining children of
    this min position.
  - β-pruning: the β value of a min position is defined to be the
    maximum possible value for that position. If the value of a max
    position is determined to be greater than or equal to the α value of
    its parent, then we may stop generation of the remaining children of
    this max position.
  - αβ-pruning: combine both α- and β-pruning. limits the searching to
    only \( O(N^{1/2}) \) nodes, where \( N \) is the size of the full game tree.
    Searches using αβ-pruning can go twice as deep as compared to an
    un-pruned tree.
  - Exploiting symmetry: avoid to recomputed symmetric solution

**Reconstruction Problem**
- Given \( N \) points located on the x-axis, we can construct the bag of
  distances between all points \( D \) in \( O(N^2) \) because \( |D| = N(N - 1)/2 \). Sort
  the distance: \( O(N^2 \log N) \)
- The turnpike reconstruction problem: reconstruct the points from the
  distances bag.
- Backtracking to solve the reconstruction problem

**Bandwidth Minimization**
- Input a graph \( G \), the goal is to find a permutation of the vertices on a
  line that minimize the maximum length of any edge.
- For a complete binary tree with 15 nodes, the pretty layout \{10, 11, 5, 9, 8, 4, 2, 1, 3, 6, 12, 13, 7, 14, 15\} has a longest edge of length 4, but
  a seemingly cramped layout \{4, 8, 2, 9, 5, 1, 10, 11, 3, 6, 12, 7, 13, 14, 15\} realizes the optimal bandwidth of 3.

**Games**
- A computer might use to play strategic game. The strategy requires the
  programmer, but not the computer, to do most of the thinking.
- Game tree: each node of the tree represents a board position, and the
  root is the starting position
- A successor position of \( P \) is any position \( P_s \) that is reachable from \( P \) by
  playing one legal move.
- Terminal position: leaves of the game tree
- Evaluation function: quantify the "goodness" of a position. A position that
  is a win for a computer gets the value +1; a draw gets a 0; and a lost
  gets a -1.
- Minimax strategy: if a position is not terminal, the value of the position is
  determined by recursively assuming optimal play by both sides. One
  player is trying to minimize the value of the position, while the other
  player is trying to maximize it.
- For more complex games, it is obviously infeasible to search all the way
  to the terminal nodes. In this case, we have to stop the search after a
  certain depth of recursion is reached.
- Ply: the number of moves of look-ahead, and it is equal to the depth of the
  recursion.
- To increase the look-ahead factor by evaluating fewer nodes without
  losing any information. Use a table to keep track of all position that have
  been evaluated, thus the second occurrence of a position need not be
  recomputed. The table is called transposition table and implemented by
  hashing.

**Combine Backtracking With Other Algorithms**
- backtracking with greedy method: the "show me the money" problem
- backtracking with randomized algorithm: the 8 queens problem