Programming Lab #8a
Arithmetic with Reals

Topics: Alternative representations of real numbers; floating-point hardware and emulation, Q16 fixed-point, posit emulation. Comparing relative performance.

Prerequisite Reading: Chapters 1-11
Revised: March 22, 2020

Background: Real numbers may be represented in floating-point, fixed-point, or posit format. Arithmetic using these representations may be implemented in hardware, or in software written in either a high-level language or assembly. This lab explores the relative performance of these alternatives by evaluating Taylor series approximations of the sine function — i.e., polynomials with coefficient \((a_0, a_1, a_2, \ldots)\) chosen to produce \(\sin(x)\):

\[
poly(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_{n-1} x^{n-1}
\]

The polynomial is most efficiently evaluated using Horner’s method\(^4\), working backwards from \(a_{n-1}\) to \(a_0\):

\[
poly(x) = (((0) x + a_{n-1}) x + a_{n-2}) x + \cdots + a_0
\]

Assignment: Create a single assembly language file containing five functions that each evaluate the polynomial shown above. The five function prototypes share a common format, but the function-name and data-type vary. (Note: The code for each of the last three functions should be almost identical.)

\[
data-type \ function-name(data-type x, data-type a[], int32_t n);
\]

<table>
<thead>
<tr>
<th>function-name</th>
<th>data-type</th>
<th>same as</th>
<th>Implement this function in assembly using ...</th>
</tr>
</thead>
<tbody>
<tr>
<td>FPH_Poly</td>
<td>float</td>
<td>float</td>
<td>floating-point addition &amp; multiply instructions</td>
</tr>
<tr>
<td>Q16_Poly</td>
<td>Q16</td>
<td>int32_t</td>
<td>integer addition &amp; multiply instructions</td>
</tr>
<tr>
<td>FPA_Poly</td>
<td>float32_t</td>
<td>int32_t</td>
<td>ASM library functions qfp_fadd &amp; qfp_fmul</td>
</tr>
<tr>
<td>FPC_Poly</td>
<td>float32_t</td>
<td>int32_t</td>
<td>C library functions AddFloats &amp; MulFloats</td>
</tr>
<tr>
<td>POS_Poly</td>
<td>posit32_t</td>
<td>int32_t</td>
<td>C library functions AddPosits &amp; MulPosits</td>
</tr>
</tbody>
</table>

The library functions are in real-libs.zip on the textbook website. Inside the zip are files lib1-float.s, lib2-float.c, and lib3-posit.c to be extracted into your src directory together with the C main program found here. The program calls each of your polynomial functions with an array of \(n\) coefficients chosen to approximate the \(\sin\) function at an angle \(x\) expressed in radians. Since the approximation of the sine function requires fewer terms near \(x = 0\) for the same accuracy, the main program varies \(n\) from 0 to 8 to reduce average execution time. The values returned by your polynomial functions are used to display moving sine waves. If your code is correct, the display should look like the image above although with possibly different cycle counts. Error messages (if any) will appear as white text on a red background.

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