Electric Circuits I
Laboratory 1: Vectors and Matrices MATLAB

Objective:

- In this laboratory you will learn how to represent and use vectors and matrices in MATLAB.

A. Background notes:

In MATLAB an $M \times N$ matrix is a rectangular array of numbers with M rows and N columns. It can be defined by typing the matrix elements between square brackets. Elements along a row are separated by commas, and rows are separated by semi colons.

In Lab 0, you used row vectors for values of the angle and time variable. A row vector is a matrix with M1.

In MATLAB Help, go to the Mathematics chapter. The second section is “Matrices in MATLAB.” Read the first four sections about creating, adding, and multiplying matrices.

B. Matrix Multiplication:

- Create 2 x 2 matrices A and B are as follows:

  A = [2,1; 3,2] and **B=[3,1; 2,2]**

- Print A’ and B’, the transposes of these two matrices.

- Compute the following 4 matrix products and print them. Are any the same? Which ones?
  A1 = A * B, A2 = B * A, A3 = (A’* B’)’, A4 = (B’ * A’)’

C. Matrix Inverses:

- Use “inv” to compute the following matrix inverses:
  A1 = inv(A * B), A2 = inv(A) * inv(B), A3 = inv(B * A), A4 = inv(B) * inv(A).

- Check the inverse values. Multiply A1*(A*B) and also multiply (A*B)*A1. What are the two products?

D. Solving Circuits with MATLAB:

The result of a KVL/KCL analysis of a circuit is the set of simultaneous equations:

$$\begin{aligned} V_1 \quad\quad\quad\;\; + V_3 &= 10 \\ 3V_1 + 3V_2 + 4V_3 &= 12 \\ 2V_1 + 2V_2 + 3V_3 &= 5 \end{aligned}$$

which can be written using matrices as follows:

$$\begin{bmatrix} 1 & 0 & 1 \\ 3 & 3 & 4 \\ 2 & 2 & 3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 12 \\ 5 \end{bmatrix} = C * V = S$$

$$C = \begin{bmatrix} 1 & 0 & 1 \\ 3 & 3 & 4 \\ 2 & 2 & 3 \end{bmatrix}, \quad V = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}, \quad S = \begin{bmatrix} 10 \\ 12 \\ 5 \end{bmatrix}$$

To solve this 3x3 system of equations we invert the coefficient matrix **C** and multiply it by the source matrix **S**.

- Invert the matrix C and solve the system for the voltage matrix V. Then verify that the result is correct by multiplying C*V and compare the result with S.

E. Solving a System of Linear Equations with Symbolic Toolbox

MATLAB or Octave has a toolbox to simplify symbolically equations. As shown in Part D, in order to solve a system of linear equations, the coefficients of equations are simplified and arranged in the order of the unknown (V_1, V_2, V_3, …, V4). Once equations are simplified and arranged in order of the known, we can generate coefficient matrix **C** and source vector **S** to solve for the unknown. The simplifying process of coefficients, however, can be tedious with chances of errors. Here, the symbolic tool performs the simplifying process and generate automatically coefficient matrix **C** and source vector **S**. Consider the following equations:

$$\frac{v_a - v_b}{R_1} - I_{s1} + \frac{v_a - v_d}{R_2} - I_{s3} = 0$$

$$\frac{v_b - v_a}{R_1} + I_{s1} + \frac{v_d - v_a}{R_2} + \frac{v_b}{R_3} + I_{s2} + \frac{v_d}{R_4} = 0$$

$$v_b - v_c = V_{s1}$$

$$v_d - v_c = V_{s2},$$

where v_a, v_b, v_c and v_d are unknown variables and others are known with values: V_{s1}= 6V, V_{s2} = 12V, I_{s1} = 2mA, Is2 = 4mA, I_{s3} = 6mA, R_1 = 1kΩ, R_2 = 1kΩ, R_3 = 2kΩ, and R_4 = 1kΩ.

- Create a command script file (.m) using the method shown in Lab0.
- Enter the values for known variables:

```
Vs1 = 6;
Vs2 = 12;
% Is1 = 2mA
Is1 = sym(2)/1e3;
…
R1  = 1e3;
```

…;

- Define symbolic unknown variables:

  ```
  syms Va Vb Vc Vd real
  ```

- Enter equations above:

  ```
  eq1 = (Va-Vb)/R1 – Is1 … == 0;
  eq2 = … == 0;
  eq3 = … == Vs1;
  eq4 = … == Vs2;
  ```

- Enter equations and variables:

  ```
  eqns = [eq1, eq2, eq3, eq4];
  vars = [Va, Vb, Vc, Vd];
  ```

- Use MATLAB or Octave functions to simplify coefficients and generate coefficient matrix **C** and source vector **S:**

  ```
  [C, S] = equationsToMatrix(eqns, vars)
  ```

- Use the coefficient matrix **C** and the source vector **S** to solve for the unknown variables:

  ```
  v = inv(C) * S
  ```

- Save the command script file and run it. If there is no error in the command script, the final results will be:

C = [1/500, -1/1000, 0, -1/1000] [-1/500, 3/2000, 0, 1/500] [0, ·1, -1, 0] [0, 0, -1, 1]	S = 1/125 -3/500 6 12	v = 13/3 -8/3 -26/3 10/3

- Create another command script file (.m) and use similar commands to solve for a system of equations below:

$$-I_{s1} + \frac{v_a - v_b}{R_1} + \frac{v_d - v_c}{R_2} + \frac{v_d}{R_5} + \frac{v_d}{R_3 + R_6} = 0$$

$$\frac{v_b - v_a}{R_1} + I_{s2} + \frac{v_c}{R_4} + \frac{v_c - v_d}{R_2} = 0$$

$$v_c - v_b = V_s$$

$$v_d - v_a = 2v_c$$

where v_a, v_b, v_c and v_d are unknown variables and others are known with values: V_s = 12V, I_{s1} = 4mA, I_{s2} = 2mA, R_1 = 1kΩ, R_2 = 1kΩ, R_3 = 1kΩ, R_4 = 1kΩ, R_5 = 1kΩ, and

$R_6 = 1k\Omega$.

- Compare your results with the expected results:

C =	S =	v =
[1/1000, -1/1000, -1/1000, 1/400] [1/1000, -1/1000, 1/500, -1/1000] [0, -1, 1, 0] [-1, 0, -2, 1]	1/250 -1/500 12 0	-100/7 2 14 96/7

F. More About Matrix Inverses:

- Find the inverse of the matrix D shown below:

D = [2,4; 1,2]

a. Is there an inverse of D?

b. If the answer is 'NO' then why not?

G. Products of Time Functions:

Given the following functions,

$$p(t) = 5\cos(2\pi 3t) \text{ and } v(t) = 5 * e^{-0.5t}$$

- plot p(t) and v(t) from t = 0 to t=10 using time steps of 0.01 (*<u>Note</u>: e^t is encoded as* $exp(t)$).

Create the point by point product function b(t) = p(t)v(t) using b=p.*v and plot it. Note that the **.*** operation multiplies the two vectors point by point rather than computing a matrix product.

<u>Note</u>: Please show all your work.