ELEN 162
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Homework 4

1. Consider the bipartite graph shown in Fig. 1.

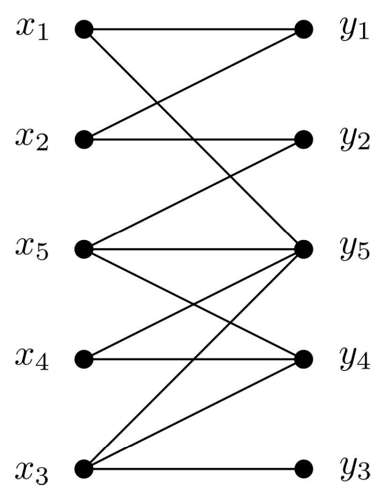

Figure 1: The bipartite graph that corresponds to matrix A.

   (a) Find a matrix A that corresponds to this graph. Is this matrix symmetric?

   (b) Does the matrix that you obtained in part (a) have a structure that could be useful for solving system

$$Ax = b \tag{1}$$

   in parallel? If so, how many processors would you use? Explain.

2. Given matrix

$$A = \begin{bmatrix} * & 0 & 0 & * & 0 & 0 \\ 0 & * & 0 & 0 & 0 & * \\ 0 & 0 & * & 0 & * & 0 \\ * & 0 & 0 & * & 0 & 0 \\ 0 & 0 & * & 0 & * & 0 \\ 0 & * & 0 & 0 & 0 & * \end{bmatrix} \tag{2}$$

   (a) Find a bipartite graph that corresponds to it.

   (b) Does the graph that you obtained in part (a) indicate that A has a structure that is suitable for parallel processing? If so, indicate how A should be permuted, and how many processors should be utilized for solving system

$$Ax = b \tag{3}$$

   If not, explain.

3. Suppose you are given a matrix whose elements are

$$A = \begin{bmatrix} 2 & 0.1 & 0 & 0.7 & 0 & 0.5 \\ 0 & 1 & 0.25 & 0.1 & 0.4 & 0 \\ 0.06 & 1 & 4 & 0 & 0.5 & 0 \\ 0.3 & 0 & 0.05 & 1 & 0.1 & 0.02 \\ 0 & 0.5 & 0.19 & 0 & 1 & 0.1 \\ 0.18 & 0.08 & 0 & 0.4 & 0 & 1 \end{bmatrix} \tag{4}$$

(a) Use epsilon decomposition to find a permutation that transforms A into a matrix with two weakly coupled diagonal blocks of dimension 3×3. Indicate the corresponding value of ε, and show the permuted matrix explicitly.

(b) If we use the permuted matrix obtained in part (a) and the Jacobi method to solve system

$$Ax = b \tag{5}$$

iteratively, how many iterations will it take to reduce the initial error

$$e(0) = \|x(0) - x^*\|_2 \tag{6}$$

by a factor of 100? (**Hint:** Think how Theorem 10.1 can help you answer this question).

(c) Repeat parts (a) and (b) assuming that the objective is to transform A into a matrix with *three* weakly coupled diagonal blocks of dimension 2×2.

4. For the matrix shown below

$$A = \begin{bmatrix} * & * & * & 0 & * & 0 & 0 \\ * & * & * & 0 & * & 0 & 0 \\ * & * & * & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & * & 0 & 0 & * \\ * & * & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & * & 0 & * & * \end{bmatrix} \tag{7}$$

(a) Draw the elimination graph and identify where the fill-in elements will occur.

(b) Perform a minimal degree ordering and identify where the fill-in elements are located in this case. Show the permuted matrix, and comment on whether the structure you obtained might be suitable for parallel processing.

5. Given the matrix that you considered in Problem 4:

(a) Use the nested dissection method to permute the matrix into a BBD form with two diagonal blocks. Show the permutation vector and the resulting matrix explicitly.

(b) Repeat part (a) of this problem using eigenvectors of the Laplacian matrix. Is the result any better than what you obtained using nested dissection? Explain.

6. For the matrix shown below

$$A = \begin{bmatrix} * & * & 0 & 0 & 0 & * & 0 & 0 & 0 \\ 0 & * & * & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & * & * & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & * & 0 & * & * & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & * & * & * & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 & * & * & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & * & * & 0 & * \\ 0 & 0 & 0 & 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & * & 0 & * \end{bmatrix} \tag{8}$$

(a) Estimate the number of fill-in elements that would arise if we were to perform an LU factorization. Why can't we determine that number precisely in this case? Explain.

(b) Set all the diagonal elements in A to 10, and the off-diagonal ones to -1. Perform an LU factorization of this matrix in Matlab and record the number of fill-ins. How well does this match the estimate that you obtained in part (a)?

(c) Permute matrix A using the minimal degree ordering, and estimate the number of fill-in elements that would arise if we factorize the permuted matrix. Show this matrix explicitly, as well as the permutation vector.

(d) Repeat part (b) for the permuted matrix that you obtained in part (c).

7. Given the matrix that you considered in Problem 6:

(a) Use the nested dissection method to permute the matrix into a BBD form with two diagonal blocks. Show the permutation vector and the resulting matrix explicitly.

(b) Repeat part (a) of this problem using eigenvectors of the Laplacian matrix. Is the result any better than what you obtained using nested dissection? Explain.