ELEN 161
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Project 3: Cellular Automata and Boolean Networks

Problem 1. In this problem (and the one that follows), you will examine the dynamic behavior of cellular automata. In particular, you will be asked to determine how Rules 30, 62 and 110 are affected by changes in L (which represents the number of cells in each row), and perturbations in the initial conditions.

(a) For each of these three rules, identify all the limit cycles (and their basins, if they happen to be attractive). Do this for $L = 4$, 5 and 6. For $L = 4$ and 5, you should represent your results in the form of schematic diagrams. In the case when $L = 6$, it will suffice to list all the limit cycles, and identify whether or not they are attractive. How sensitive is the number and average length of the attractive cycles to changes in L? Explain.

(b) Consider Rule 110 and initial conditions 1, 3, 7 and 15 (each of which can be obtained from the previous one by a single "bit flip"). Test this rule for all four initial conditions and $L = 4$, 5 and 6, and record how the system evolves in each case. Based on your results, how sensitive would you say this rule is to small perturbations in the initial conditions?

Problem 2. Use a cellular automata generator (I suggest the one at http://www.cellularautomatagenerator.com/) to examine how Rules 30, 62 and 110 behave when L is large. Simulate the system behavior for four initial conditions that differ by a single bit, as in Problem 1(b) (you can choose these initial conditions yourself). Print out the patterns that you obtain in each case, and use these results to evaluate how sensitive Rules 30, 62 and 110 are to small perturbations in the initial conditions when L is large. Are your conclusions consistent with the ones you reached in Problem 1(b)?

Problem 3. Consider the Boolean network shown in Fig. 1, in which $N = 4$ and $K = 2$. It is assumed that the dynamics of node i in this network are governed by equation

$$x_i(k+1) = F_i[x(k)] \tag{1}$$

where functions F_i $(i = 1, 2, 3, 4)$ are defined as

x_2	x_4	$F_1(x_2, x_4)$
0	0	0
0	1	1
1	0	0
1	1	1

x_1	x_4	$F_2(x_1, x_4)$
0	0	1
0	1	1
1	0	1
1	1	0

(2)

x_1	x_2	$F_3(x_1, x_2)$
0	0	0
0	1	0
1	0	1
1	1	1

x_1	x_3	$F_4(x_1, x_3)$
0	0	1
0	1	0
1	0	0
1	1	1

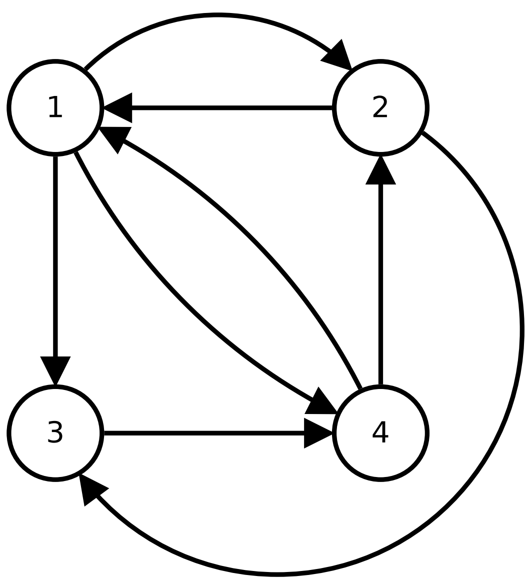

Figure 1: Boolean network with $N = 4$ and $K = 2$.

(a) Find all the limit cycles and basins of attraction for this system. Represent your results schematically (as in Problem 1(a)), and identify which cycles are attractive.

(b) Consider the following sequence of single bit flips in the functions F_i defined in (2) (the bit that has changed is framed by a box).

x_2	x_4	$F_1(x_2, x_4)$
0	0	0
0	1	1
1	0	0
1	1	$\boxed{0}$

x_1	x_4	$F_2(x_1, x_4)$
0	0	1
0	1	$\boxed{0}$
1	0	1
1	1	0

x_1	x_2	$F_3(x_1, x_2)$
0	0	0
0	1	$\boxed{1}$
1	0	1
1	1	1

x_1	x_3	$F_4(x_1, x_3)$
0	0	$\boxed{0}$
0	1	0
1	0	0
1	1	1

(3)

In each case, determine the number and average length of attractive limit cycles. As before, represent your results in the form of schematic diagrams, and use this information to evaluate the sensitivity of the system to small perturbations in F_i.

Problem 4. Repeat Problem 3 for a network with $N = 4$ and $K = 3$, whose connectivity matrix is

$$U = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 3 & 4 \\ 1 & 2 & 4 \\ 1 & 2 & 3 \end{bmatrix} \tag{4}$$

In this case, assume that functions F_i are the ones given in equation (5).

Note. In part (b) of this problem, you can choose your own set of single bit flips in functions $F_1 - F_4$. Make sure, however, to explictly indicate what they are.

x_2	x_3	x_4	$F_1(x_2, x_3, x_4)$
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

x_1	x_3	x_4	$F_2(x_1, x_3, x_4)$
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

(5)

x_1	x_2	x_4	$F_3(x_1, x_2, x_4)$
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

x_1	x_2	x_3	$F_4(x_1, x_2, x_3)$
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0