ELEN 161
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Homework 2

1. Suppose that a quantum particle is in a state of superposition, which is characterized by wave function

$$\psi(x) = 0.63\,\psi_0 + 0.5\,\psi_1 + 0.45\,\psi_2 + 0.388\,\psi_3$$

If we assume that states ψ_0, ψ_1, ψ_2 and ψ_3 correspond to different energy levels, find the probabilities for each of the four possible values that we may record when we perform a measurement.

2. Consider a single qubit whose wave function has the form $\psi = \alpha_0\psi_0 + \alpha_1\psi_1$. Find α_0 and α_1 so that the state of the qubit remains *unchanged* after it passes through a Hadamard gate. **Note:** Make sure that function ψ is normalized.

3. Consider a pair of qubits whose wave functions are $\psi_a = \alpha_0\psi_0 + \alpha_1\psi_1$ and $\psi_b = \beta_0\psi_0 + \beta_1\psi_1$, respectively. How should functions ψ_a and ψ_b be chosen so that their tensor product $\psi_a \otimes \psi_b$ remains *unchanged* after passing through a C-NOT gate? Is your answer unique? Explain.

4. Consider the system shown in Fig. 1, which consists of a Z gate, a Hadamard gate and a C-NOT gate.

(a) Find the output function Φ.

(b) Based on the answer you obtained in part (a), what is the probability that the two qubits will be found in the *same* state when we perform a measurement?

(c) What is the connection between your answer in part (b) and Einstein's famous EPR paradox? Explain.

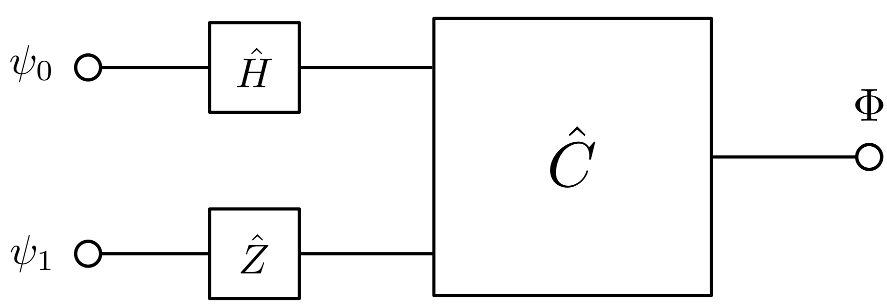

Figure 1: The system for Problem 4.

5. Consider the system shown in Fig. 2, which consists of a Z gate, two Hadamard gates, and a C-NOT gate. Function ψ_X is assumed to have the form

$$\psi_X = 0.5774\,\psi_0 + 0.8165\,\psi_1$$

and $\psi_Y = \psi_0$.

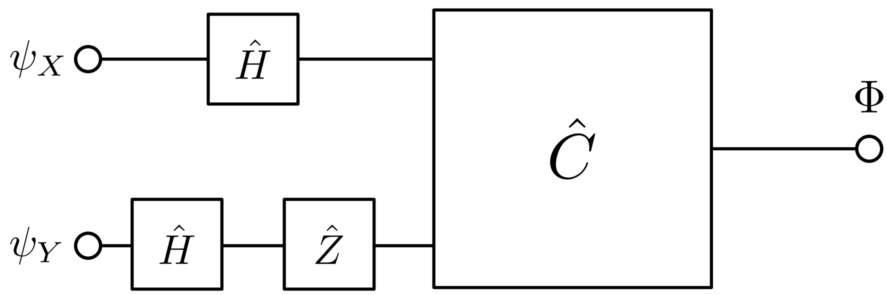

Figure 2: The system for Problem 5.

(a) Find the output function Φ.

(b) Using the results obtained in part (a), calculate the probabilities for each of the four possible outcomes that we can register if we perform a measurement on both qubits. How likely is it that we will find the control qubit in state ψ_1?

6. Consider the system shown in Fig. 3, which consists of a Z gate, a Hadamard gate, and a Toffoli gate.

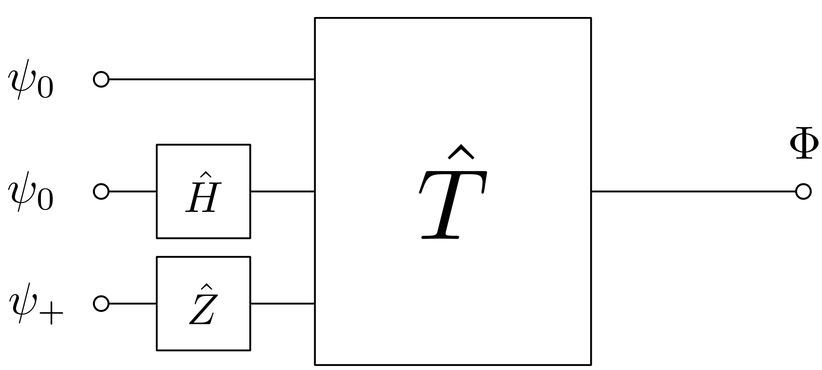

Figure 3: The system for Problem 6.

(a) Find the output function Φ.

(b) Based on the results obtained in part (a), calculate the probabilities for each of the 8 possible outcomes that we can register if we perform a measurement on all three qubits. Which triplet of states are we most likely to see?

(c) Are there any combinations of states that we cannot possibly encounter? Explain.

(d) What role does the Toffoli gate play in this particular system? Would you say that it performs a useful function? Explain.