ELEN 161
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Homework 4

1. Consider the sand pile configuration shown in Fig. 1, in which the critical height is assumed to be 4.

0	1	2	2
2	3	3	1
2	3	2	2
0	2	1	2

Figure 1: The initial sand pile configuration.

(a) Examine what happens when an additional grain of sand is added to the three locations highlighted in gray. Show the sequence of toppling events explicitly for each case.

(b) If we view the number of locations where the height equals 3 as an indicator of how close the system is to an avalanche, would you say that any of the three relaxation processes examined in part (a) decrease the likelihood of such an event? Explain.

2. Consider a cellular automaton with 3 cells, whose dynamics are governed by the rule shown in Table 1. You can assume that a "wraparound" scheme is in place where x_3 acts as the "left neighbor" of x_1, and x_1 represents the "right neighbor" of x_3.

$x_{i-1}(k)$	$x_i(k)$	$x_{i+1}(k)$	$x_i(k+1)$
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Table 1. Rule 106 for the cellular automaton.

(a) Construct a matrix M that relates the states of the system in steps k and $k+1$.

(b) Use this matrix to determine all the limit cycles in the system.

(c) For each cycle, indicate whether it is an attractor or an "Isle of Eden". If it is the former, specify its basin of attraction as well.

(d) Use the results obtained in parts (a)-(c) to sketch the temporal evolution of the automaton for the following two initial conditions:

(a) (b)

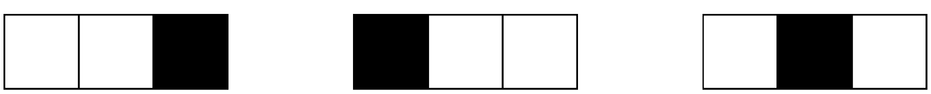
Figure 2: Initial conditions for Problem 2.

3. Repeat Problem 2 for an automaton whose dynamics are described by the following table:

$x_{i-1}(k)$	$x_i(k)$	$x_{i+1}(k)$	$x_i(k+1)$
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

Table 2. Rule 219 for the cellular automaton.

In part (d), sketch the temporal evolution of the automaton for the following three initial conditions:



(a) (b) (c)

Figure 3: Initial conditions for Problem 3.

4. Consider a Boolean network with $N = 3$ and $K = 2$, where functions $F_1[x(k)]$, $F_2[x(k)]$ and $F_3[x(k)]$ are defined by Tables 3-5:

(a) Construct matrix M that relates the states of the system in steps k and $k+1$.

(b) Use this matrix to determine all the limit cycles in the system.

(c) For each cycle, indicate whether it is an attractor or an "Isle of Eden". If it is the former, also specify its basin of attraction.

(d) Once the network settles into its attractive limit cycle, will any of its nodes be fixed (i.e. "frozen")? Explain.

$x_2(k)$	$x_3(k)$	$x_1(k+1)$
0	0	1
0	1	1
1	0	1
1	1	1

Table 3. Function $F_1\left[x(k)\right]$.

$x_1(k)$	$x_3(k)$	$x_2(k+1)$
0	0	1
0	1	0
1	0	1
1	1	0

Table 4. Function $F_2\left[x(k)\right]$.

$x_1(k)$	$x_2(k)$	$x_3(k+1)$
0	0	0
0	1	0
1	0	0
1	1	1

Table 5. Function $F_3\left[x(k)\right]$.

5. Repeat Problem 4 for a Boolean network with $N = 3$, whose dynamics are governed by the following functions:

$x_1(k)$	$x_2(k)$	$x_1(k+1)$
0	0	0
0	1	1
1	0	1
1	1	0

Table 6. Function $F_1\left[x(k)\right]$.

$x_1(k)$	$x_3(k)$	$x_2(k+1)$
0	0	1
0	1	0
1	0	0
1	1	0

Table 7. Function $F_2\,[\,x(k)]$.

$x_3(k)$	$x_3(k+1)$
0	1
1	0

Table 8. Function $F_3\,[\,x(k)]$.

Note that this network is somewhat different from the ones that Kauffman used in his simulations, because K is *not* the same for all nodes. It is also helpful to keep in mind that $x_3(k+1)$, depends *only* on $x_3(k)$, and that $x_1(k+1)$ depends on $x_1(k)$ (in addition to $x_2(k)$).