ELEN 161
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Homework 3

1. Consider the linear system

$$\dot{x}_1 = -2px_1 + qx_2$$
$$\dot{x}_2 = -qx_1 - pqx_2$$

where p and q are unspecified parameters. How should p and q be chosen to ensure that the solutions of this system have the form

$$x_1(t) = ae^{-t} + be^{-2t}$$
$$x_2(t) = ce^{-t} + de^{-2t}$$

for any choice of initial conditions? Is your answer unique? Explain.

2. Consider the nonlinear system

$$\dot{x} = x^3 + (p+1)x^2 + px$$

where p is a parameter.

(a) Determine all the equilibria as a function of p.

(b) For each equilibrium, determine the range of p for which it is stable.

(c) If parameter p is varied from -5 to 5, the properties of the equilibria will change, but one of them will always be stable. Plot this equilibrium as a function of p, and determine whether there are any discontinuities.

(d) Based on the answer you obtained in part (c), is it possible to encounter a situation where the stable equilibrium changes abruptly as a result of a small perturbation in p? Is it possible, in other words, for this system to experience a "catastrophe"? Explain.

3. Repeat Problem 2 for system

$$\dot{x} = x^3 - 2p^2x^2 - px + 2p^3$$

Does a stable equilibrium exist for all values of p?

4. Consider the system

$$\dot{x} = -x^3 + 2.5x - p$$

where p is initially fixed at $p = 1.5$.

(a) Find all the equilibria of this system, and determine their stability properties.

(b) Suppose the system is in one of the stable equilibria that you identified in part (a). What will happen if p is suddenly increased by 1.5%? And to what extent does your answer depend on which equilibrium the system was in prior to the perturbation? Explain.

(c) How is the behavior of this system different from the ones in Problems 2 and 3? Explain.