

ELEN 160
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Homework 3

PROBLEM 1. Consider the system

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_1 + p(1 - x_2^2)x_2\end{aligned}$$

where p represents a parameter.

- Show that $x_1 = x_2 = 0$ is the *unique* equilibrium for all p . Determine how the stability properties of this equilibrium change in the interval $-5 \leq p \leq 5$.
- Solve the system numerically for: $p = -3$, $p = -1$, $p = 0.5$ and $p = 3$. In each case, provide a phase plot using at least three different initial conditions.
- Based on the results obtained in Part b), determine what kind of bifurcation occurs at $p = 0$.

PROBLEM 2. The system shown below

$$\begin{aligned}\dot{x}_1 &= -x_1 - x_1^2 + x_2 \\ \dot{x}_2 &= px_1 - x_2 - x_2^2\end{aligned}$$

has two equilibria for all values of p .

- Find a value p^* for which the two equilibria coincide.
- Show that for $p = p^*$ the system experiences a *transcritical* bifurcation.

NOTE. This will require examining the stability of the Jacobian in the neighborhood of p^* .

PROBLEM 3. The system

$$\begin{aligned}\dot{x}_1 &= -x_1^2 + x_2 \\ \dot{x}_2 &= (p^2 + 2p + 3)x_1 - px_2\end{aligned}$$

has two equilibria - one at the origin, and a second one that changes with p .

- Obtain an explicit expression for the “moving” equilibrium $x^e(p)$ (note that this equilibrium does not exist for $p = 0$).
- Show that $x^e = 0$ is *unstable* for all p .
- Show that $x^e(p)$ is unstable for $p < 0$ and stable for $p > 0$. You can do this by plotting the eigenvalues of the Jacobian, $\lambda_1(p)$ and $\lambda_2(p)$, for $-10 \leq p \leq -0.1$ and $0.1 \leq p \leq 10$.