ELEN 160
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Homework 1

PROBLEM 1. For the system shown below

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ -2 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

a) Find the modal decomposition (without using Matlab).

b) Plot the analytic solution obtained in Part a), and compare it with the numerical solution generated by Matlab. Do you see any difference between the two?

PROBLEM 2. For the system shown below

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 6 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

a) Compute the modal decomposition (again, without Matlab), and determine the conditions that a stable manifold must satisfy.

b) For initial conditions $x_1(0) = 1$ and $x_2(0) = 2$ (which happen to be on the stable manifold), plot the analytic solution $x_1(t)$.

c) Use Matlab to solve the system numerically on the interval $t \in [0, 20]$ with the same initial conditions as in Part b). Plot the resulting function $x_1(t)$, and compare it with the graph obtained in Part b).

d) Repeat the simulation performed in Part c), this time for the interval $t \in [0, 40]$. Compare your results with the plot obtained in Part b), and explain any discrepancies.

PROBLEM 3. Consider the system

$$\dot{x} = Ax$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -12 & -4 & 15 & 5 & -3 \end{bmatrix}$$

a) Determine the modal decomposition for this system, and use it to find the conditions for a stable manifold.

b) If you are told that $x_3(0) = 1$, $x_4(0) = 0$ and $x_5(0) = 0$, find values for $x_1(0)$ and $x_2(0)$ for which $x(t) \to 0$ when $t \to \infty$. What does $x(t)$ look like for this choice of initial conditions? Write this solution explicitly.

PROBLEM 4. In this problem, you are told that the solution of system

$$\dot{x} = Ax$$

has the form

$$x_1(t) = e^{-2t} + 2e^{-4t} + e^{t}$$
$$x_2(t) = 2e^{-4t} + 2e^{t}$$
$$x_3(t) = e^{-2t} + e^{-4t} + e^{-0.5t} + e^{t}$$
$$x_4(t) = e^{-2t} + 2e^{t}$$

a) Determine matrix A and the initial conditions that correspond to this solution.

b) Find a set of initial conditions for which *only* e^{-4t} appears in the solution. Show $x(t)$ explicitly for one such choice of initial conditions.