ELEN 118
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HOMEWORK # 1

1. For the matrix below:

$$A = \begin{bmatrix} * & * & * & 0 & * & 0 & 0 \\ * & * & * & * & * & 0 & 0 \\ * & * & * & 0 & 0 & 0 & 0 \\ 0 & * & 0 & * & 0 & 0 & 0 \\ * & * & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$

a) Draw the elimination graph and identify where the fill in elements will occur.

b) Perform a minimal degree ordering and identify fill in elements in this case. Show the permuted matrix.

c) Use the nested dissection algorithm to find a minimal separator. Show the permuted matrix.

2. For the matrix below:

$$A = \begin{bmatrix} * & * & 0 & 0 & 0 & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 & 0 & 0 & 0 \\ 0 & * & * & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & * & 0 & * & * & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & * & * & * & 0 & 0 & 0 \\ * & 0 & 0 & 0 & * & * & * & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & * & 0 & * \end{bmatrix}$$

a) Draw the elimination graph and identify where the fill in elements will occur.

b) Perform a minimal degree ordering and identify fill in elements in this case. Show the permuted matrix.

c) Use the nested dissection algorithm to find a minimal separator. Show the permuted matrix.

3. For the four systems of equations shown below

a) Find matrices L and U using *both* factorization schemes

b) Solve the equations by forward and backward substitution.

SYSTEM 1

$$\begin{bmatrix} 4 & 6 \\ 2 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

SYSTEM 2

$$\begin{bmatrix} 2 & 4 & 6 \\ 6 & 4 & 2 \\ 1 & 8 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ 13 \end{bmatrix}$$

SYSTEM 3

$$\begin{bmatrix} 1 & 0 & 0 & 2 & 1 \\ 3 & 2 & 0 & 0 & 2 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

SYSTEM 4

$$\begin{bmatrix} 2 & 4 & 0 & 2 & 0 \\ 0 & 3 & 6 & 0 & 3 \\ 1 & 2 & 1 & 2 & 0 \\ 2 & 4 & 1 & 4 & 0 \\ 0 & 1 & 2 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 3 \\ 6 \\ 2 \end{bmatrix}$$