Electric Circuits II

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Project 2: Transient Analysis

The circuit below

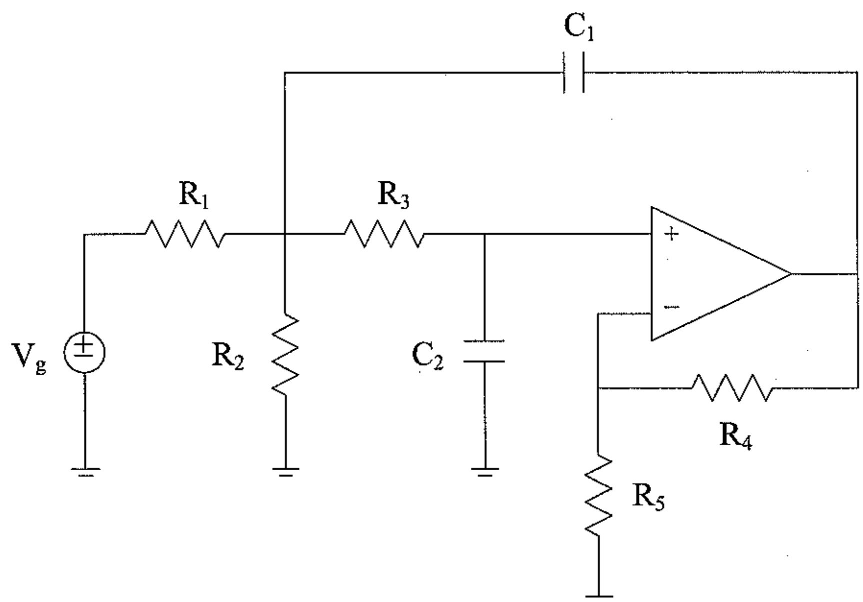

Fig. 1. A test circuit for transient analysis.

it driven by a *unit step function.*

Problem 1. Derive the differential-algebraic equations that describe this circuit (express them in matrix form).

Problem 2. Write an m-file that simulates the step response of this circuit. Use this file to obtain a plot that corresponds to the following element values: $R_1 = R_2 = 5\,K\Omega$, $R_3 = 400\,\Omega$, and $C_1 = C_2 = 0.1\,\mu F$. What is the *peak overshoot* in this case? NOTE: The peak overshoot is defined as the maximal amount by which $v_0(t)$ exceeds $1\,V$.

Problem 3. Perform a transient analysis of your circuit in SPICE, and compare with the results obtained using Matlab.

Problem 4. If the elements of the circuit in Fig. 1 are chosen as

$$R_4 = R_5 = 1\,K\Omega \tag{1}$$

$$C_1 = C_2 = 0.1\,\mu F \tag{2}$$

$$R_1 = R_2 = \frac{2}{a} \cdot 10^7 \tag{3}$$

$$R_3 = \frac{a}{b} \cdot 10^7 \tag{4}$$

it can be shown that the corresponding transfer function has the form

$$H(s) = \frac{V_0(s)}{V_g(s)} = \frac{b}{s^2 + as + b} \tag{5}$$

Use partial fraction expansion to establish that the resulting step response can be expressed as

$$v_0(t) = 1 - e^{-\alpha t} \cos \beta t - (\frac{\alpha}{\beta}) e^{-\alpha t} \sin \beta t \tag{6}$$

where $\alpha = a/2$ and $\beta = \sqrt{b - (a/2)^2}$.

Problem 5. Based on Problem 4, show that the maximal and minimal values of $v_0(t)$ occur at times t_k that satisfy $\sin \beta t_k = 0$. Use this result to show that the *first* maximum occurs at time

$$t_1 = \frac{\pi}{\beta} \tag{7}$$

and that the peak overshoot is

$$p.o. = e^{-\alpha\pi/\beta} \tag{8}$$

Problem 6. Using equations (1)-(4), design R_1, R_2, and R_3 so that the peak overshoot of the circuit in Fig. 1 is *no larger than* 10%. Use the m-file developed in Problem 2 to verify your design.

Problem 7. Assemble the circuit with the element values given in Problem 2, and measure the peak overshoot. Repeat this for the values obtained in Problem 6. In both cases, compare your measurements to the simulation results.