Power Calculations

Example 1

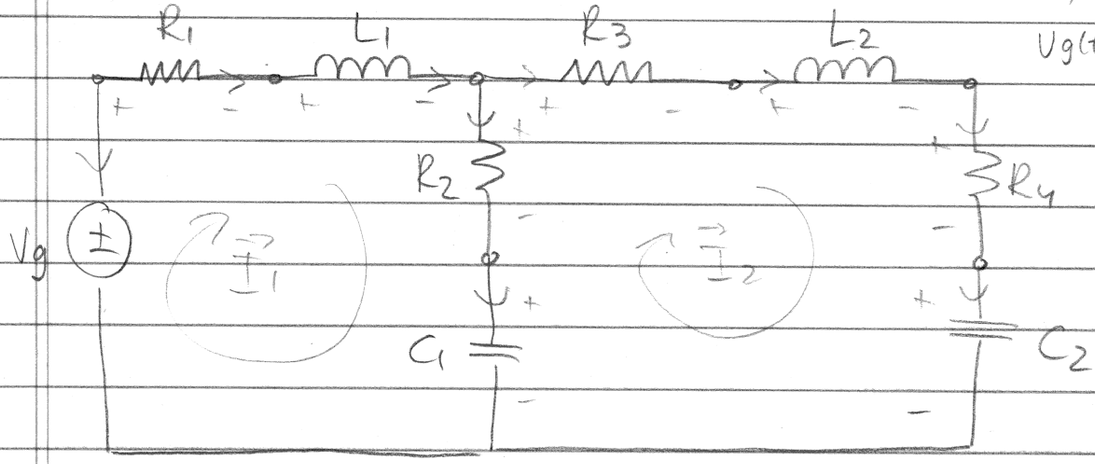
$R_1 = R_2 = R_3 = R_4 = 1\Omega$

$C_1 = C_2 = 1F$; $L_1 = 2H$

$L_2 = 1H$; $\omega = 1$

$v_g(t) = 5\sin t = 5\cos(t - 90°)$

$\vec{V}_g = 5\angle -90° = -j5$



1) $-\vec{V}_g + \vec{V}_{R1} + \vec{V}_{L1} + \vec{V}_{R2} + \vec{V}_{C1} = 0$

2) $\vec{V}_{R3} + \vec{V}_{L2} + \vec{V}_{R4} + \vec{V}_{C2} - \vec{V}_{C1} - \vec{V}_{R2} = 0$

$\vec{V}_{R1} = R_1 \vec{I}_{R1} = R_1 \vec{I}_1$

$\vec{V}_{R2} = R_2 \vec{I}_{R2} = R_2 (\vec{I}_1 - \vec{I}_2)$

$\vec{V}_{R3} = R_3 \vec{I}_{R3} = R_3 \vec{I}_2$

$\vec{V}_{R4} = R_4 \vec{I}_{R4} = R_4 \vec{I}_2$

$\vec{V}_{L1} = j\omega L_1 \vec{I}_{L1} = j\omega L_1 \vec{I}_1$

$\vec{V}_{L2} = j\omega L_2 \vec{I}_{L2} = j\omega L_2 \vec{I}_2$

$\vec{V}_{C1} = \frac{1}{j\omega C_1} \vec{I}_{C1} = \frac{1}{j\omega C_1} (\vec{I}_1 - \vec{I}_2)$

$\vec{V}_{C2} = \frac{1}{j\omega C_2} \vec{I}_{C2} = \frac{1}{j\omega C_2} \vec{I}_2$

1) $-\vec{V}_g + R_1 \vec{I}_1 + j\omega L_1 \vec{I}_1 + R_2 (\vec{I}_1 - \vec{I}_2) + \frac{1}{j\omega C_1} (\vec{I}_1 - \vec{I}_2) = 0$

2) $R_3 \vec{I}_2 + j\omega L_2 \vec{I}_2 + R_4 \vec{I}_2 + \frac{1}{j\omega C_2} \vec{I}_2 - \frac{1}{j\omega C_1} (\vec{I}_1 - \vec{I}_2) - R_2 (\vec{I}_1 - \vec{I}_2) = 0$

$1 + j2 + 1 + \frac{1}{j} = 2 + j$ ($\frac{1}{j} = -j$)

$1 + \frac{1}{j} = 1 - j$

1) $\vec{I}_1 (R_1 + j\omega L_1 + R_2 + \frac{1}{j\omega C_1}) - \vec{I}_2 (R_2 + \frac{1}{j\omega C_1}) = \vec{V}_g = -j5$

2) $-\vec{I}_1 (R_2 + \frac{1}{j\omega C_1}) + \vec{I}_2 (R_3 + j\omega L_2 + R_4 + \frac{1}{j\omega C_2} + \frac{1}{j\omega C_1} + R_2) = 0$

$1 + \frac{1}{j} = 1 - j$

$1 + j + 1 + \frac{1}{j} + \frac{1}{j} + 1 = 3 - j$

$$\begin{bmatrix} 2+j & -1+j \\ -1+j & 3-j \end{bmatrix} \begin{bmatrix} \vec{I}_1 \\ \vec{I}_2 \end{bmatrix} = \begin{bmatrix} -j5 \\ 0 \end{bmatrix}$$

$\vec{I}_1 = -1.3793 - j1.5517 \Rightarrow |\vec{I}_1| = 2.0761$

$\vec{I}_2 = -0.8621 - j0.3448 \Rightarrow |\vec{I}_2| = 0.9285$

$\vec{I}_{R2} = \vec{I}_{C1} = (\vec{I}_1 - \vec{I}_2) = -0.5172 - j1.2069 \Rightarrow |\vec{I}_{R2}| = |\vec{I}_{C1}| = 1.3131$

resistors (active power only)

$S_{R1} = \frac{1}{2}|\vec{I}_{R1}|^2 \cdot Z_{R1} = \frac{1}{2}|\vec{I}_1|^2 \cdot 1 = \frac{1}{2}\cdot(2.0761)^2 \cdot 1 = 2.155\,W$

$S_{R2} = \frac{1}{2}|\vec{I}_{R2}|^2 \cdot Z_{R2} = \frac{1}{2}(1.3131)^2 \cdot 1 = 0.8621\,W$

$S_{R3} = \frac{1}{2}|\vec{I}_{R3}|^2 \cdot Z_{R3} = \frac{1}{2}|\vec{I}_2|^2 \cdot 1 = \frac{1}{2}(0.9285)^2 \cdot 1 = 0.431\,W$

$S_{R4} = \frac{1}{2}|\vec{I}_{R4}|^2 \cdot Z_{R4} = \frac{1}{2}|\vec{I}_2|^2 \cdot 1 = 0.431\,W$

capacitors (reactive power only)

$S_{C1} = \frac{1}{2} Z_{C1} \cdot |\vec{I}_{C1}|^2 = \frac{1}{2}\frac{1}{j\omega C_1}\cdot \underbrace{(1.3131)^2}_{1.7242} = -j\frac{1}{2}\cdot 1.7242 = -j0.8621$

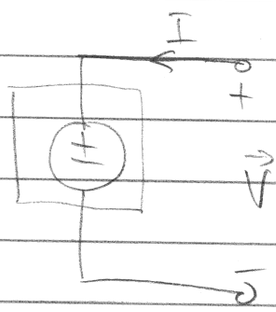
$S_{C2} = \frac{1}{2} Z_{C2} \cdot |\vec{I}_{C2}|^2 = \frac{1}{2}\cdot\frac{1}{j\omega C_2}\cdot(0.9285)^2 = -j\cdot\frac{1}{2}\cdot 0.8621 = -j0.431$

inductors

$S_{L1} = \frac{1}{2} Z_{L1} \cdot |\vec{I}_{L1}| = \frac{1}{2}\underbrace{(j\omega L_1)}_{jZ}\cdot(2.0761)^2 = j\cdot 4.31$

$S_{L2} = \frac{1}{2} Z_{L2} \cdot |\vec{I}_{L2}| = \frac{1}{2}\underbrace{(j\omega L_2)}_{j}\cdot(0.9285)^2 = j0.431$

For the source:



$$S = \frac{1}{2}\vec{V}\cdot\vec{I}^{*}$$

$$\vec{V} = \vec{V}_g = -j5 \;;\quad \vec{I} = -\vec{I}_1 = 1.3793 + j1.5517$$

$$\Rightarrow \quad S_g = \frac{1}{2}(-j5)\cdot(1.3793 - j1.5517) = -3.879 - j3.448$$

ACTIVE POWER DELIVERED $\Rightarrow$ 3.879 W ✓ (source)

TOTAL ACTIVE POWER ABSORBED: (resistors)

$2.155 + 0.8621 + 0.431 + 0.431 = 3.879$ W ✓

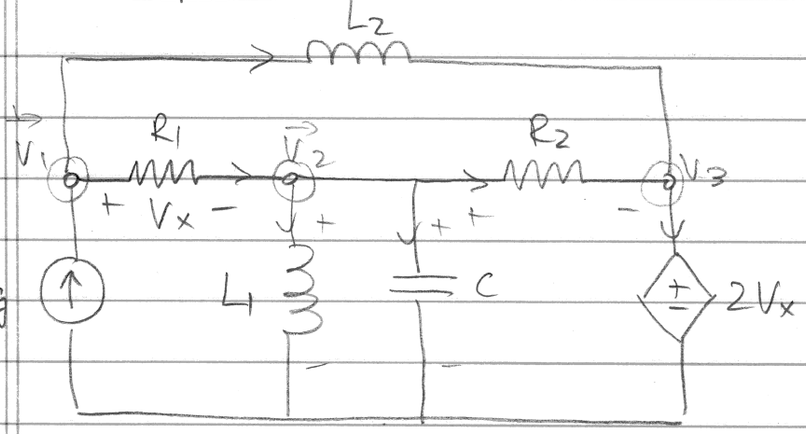
REACTIVE POWER DELIVERED: (source + capacitors)

$3.448 + 0.862 + 0.431 = 4.741$ VAR ✓

REACTIVE POWER ABSORBED: (inductors)

$4.31 + 0.431 = 4.741$ VAR ✓

Example 2



$R_1 = R_2 = 1\Omega$

$L_1 = L_2 = 1H$

$C = 4F$

$i_g(t) = \sin t = \cos(t - 90°)$

$\Downarrow$

$\vec{I}_g = 1\angle -90° = -j$

1) $-\vec{I}_g + \vec{I}_{R1} + \vec{I}_{L2} = 0$

2) $-\vec{I}_{R1} + \vec{I}_{L1} + \vec{I}_C + \vec{I}_{R2} = 0$

3) $-\vec{I}_{L2} - \vec{I}_{R2} + \vec{I}_x = 0$

$\vec{I}_{R1} = (\vec{V}_1 - \vec{V}_2)/R_1 = -1.4 - j0.2$

$\vec{I}_{R2} = (\vec{V}_2 - \vec{V}_3)/R_2 = 2.2 + j1.6$

$\vec{I}_{L1} = \vec{V}_2/j\omega L_1 = 1.2 + j0.6$

$\vec{I}_{L2} = (\vec{V}_1 - \vec{V}_3)/j\omega L_2 = 1.4 - j0.8$

$\vec{I}_C = j\omega C \vec{V}_2 = -4.8 - j2.4$

$\vec{I}_x = ?$

1) $-\vec{I}_g + \dfrac{(\vec{V}_1 - \vec{V}_2)}{R_1} + \dfrac{(\vec{V}_1 - \vec{V}_3)}{j\omega L_2}$

2) $-\dfrac{(\vec{V}_1 - \vec{V}_2)}{R_1} + \dfrac{\vec{V}_2}{j\omega L_1} + j\omega C \vec{V}_2 + \dfrac{(\vec{V}_2 - \vec{V}_3)}{R_2} = 0$

3) $\vec{V}_3 = 2\vec{V}_x = 2(\vec{V}_1 - \vec{V}_2)$

$1 + \frac{1}{j} = 1 - j$ $\qquad$ $\frac{1}{j} = -j$

1) $\vec{V}_1\left(\frac{1}{R_1} + \frac{1}{j\omega L_2}\right) - \frac{1}{R_1}\vec{V}_2 - \frac{1}{j\omega L_2}\vec{V}_3 = \vec{I}_g = -j$

$1 + \frac{1}{j} + j4 + 1 = 2 + j3$

2) $-\frac{1}{R_1}\vec{V}_1 + \vec{V}_2\left(\frac{1}{R_1} + \frac{1}{j\omega L_1} + j\omega C + \frac{1}{R_2}\right) - \frac{1}{R_2}\vec{V}_3 = 0$

3) $2\vec{V}_1 - 2\vec{V}_2 - \vec{V}_3 = 0$

$$\begin{bmatrix} 1-j & -1 & j \\ -1 & (2+j3) & -1 \\ 2 & -2 & -1 \end{bmatrix} \begin{bmatrix} \vec{V}_1 \\ \vec{V}_2 \\ \vec{V}_3 \end{bmatrix} = \begin{bmatrix} -j \\ 0 \\ 0 \end{bmatrix}$$

$\vec{V}_1 = -2+j$

$\vec{V}_2 = -0.6+j1.2$

$\vec{V}_3 = -2.8-j0.4$

resistors:

$S_{R1} = \frac{1}{2} Z_{R1} \cdot |\vec{I}_{R1}|^2 = \frac{1}{2} R_1 \cdot |-1.4-j0.2|^2 = 1.0\ W$

$S_{R2} = \frac{1}{2} Z_{R2} \cdot |\vec{I}_{R2}|^2 = \frac{1}{2} R_2 \cdot |2.2+j1.6|^2 = 3.7\ W$

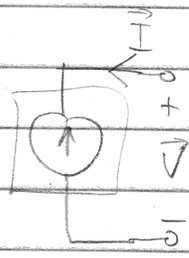
inductors:

$S_{L1} = \frac{1}{2} Z_{L1} |\vec{I}_{L1}|^2 = \frac{1}{2} j\omega L_1 |1.2+j0.6|^2 = 0.9j$ (where $j\omega L_1 = j$)

$S_{L2} = \frac{1}{2} Z_{L2} |\vec{I}_{L2}|^2 = \frac{1}{2} j\omega L_2 |1.4-j0.8|^2 = 1.3j$ (where $j\omega L_2 = j$)

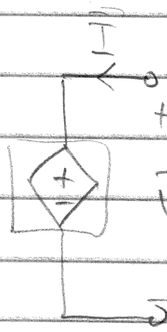
capacitor:

$S_C = \frac{1}{2} Z_C |\vec{I}_C|^2 = \frac{1}{2} \cdot \frac{1}{j\omega C} \cdot |-4.8-j2.4|^2 = -3.6j$ (where $\frac{1}{j\omega C} = -j4$)



$S_1 = \frac{1}{2} \vec{V} \cdot \vec{I}^*$ $\quad \vec{V} = \vec{V}_1 = -2+j$, $\quad \vec{I} = -\vec{I}_g = j$

$\Rightarrow S_1 = \frac{1}{2}(-2+j)(-j) = 0.5+j$



$S_2 = \frac{1}{2} \vec{V} * \vec{I}^*$ $\quad \vec{V} = \vec{V}_3 = -2.8-j0.4$, $\quad \vec{I} = \vec{I}_{R2} + \vec{I}_{L2} = 3.6+j0.8$

$S_2 = \frac{1}{2}(-2.8-j0.4) * (3.6-j0.8) = -5.2+j0.4$

ACTIVE POWER DELIVERED: 5.2 W ✓

↖ controlled source

ACTIVE POWER ABSORBED:

$1 + 3.7 + 0.5 = 5.2\,W$ ✓

↑ R_1 ↑ R_2 ↑ I_g

REACTIVE POWER DELIVERED:

3.6 VAR (capacitor)

↑ C

REACTIVE POWER ABSORBED:

$0.9 + 1.3 + 1 + 0.4 = 3.6\ VAR$

↑ L_1 ↑ L_2 ↑ I_g ↑ ctrl. source