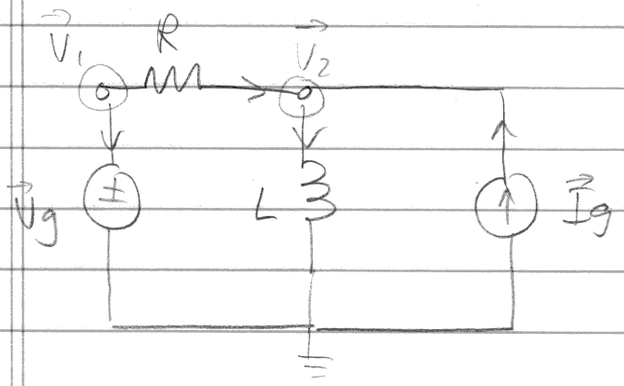
The Node Voltage Method

Example 1



$R = 1\,\Omega$; $L = 1H$

$v_g = 2\cos t \rightarrow \vec{V}_g = 2$

$i_g = 5\cos t \rightarrow \vec{I}_g = 5$

→ redraw as phasor circuit

1) $\vec{I}_{Vg} + \vec{I}_R = 0$

2) $-\vec{I}_R + \vec{I}_L - \vec{I}_g = 0$

$\vec{I}_R = (\vec{V}_1 - \vec{V}_2)/R$

$\vec{I}_L = \vec{V}_2 / j\omega L$

$\vec{I}_{Vg} = ?$

1) $\vec{V}_1 = \vec{V}_g = 2$

2) $-\frac{(\vec{V}_1 - \vec{V}_2)}{R} + \frac{\vec{V}_2}{j\omega L} - 5 = 0$

$\Rightarrow$ 1) $\vec{V}_1 = 2$

2) $-\frac{1}{R}\vec{V}_1 + \vec{V}_2\left(\frac{1}{R} + \frac{1}{j\omega L}\right) = 5$

$1 + \frac{1}{j} = 1 - j$

$$\begin{bmatrix} 1 & 0 \\ -1 & 1-j \end{bmatrix} \begin{bmatrix} \vec{V}_1 \\ \vec{V}_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix} \rightarrow \vec{V}_1 = 2$$

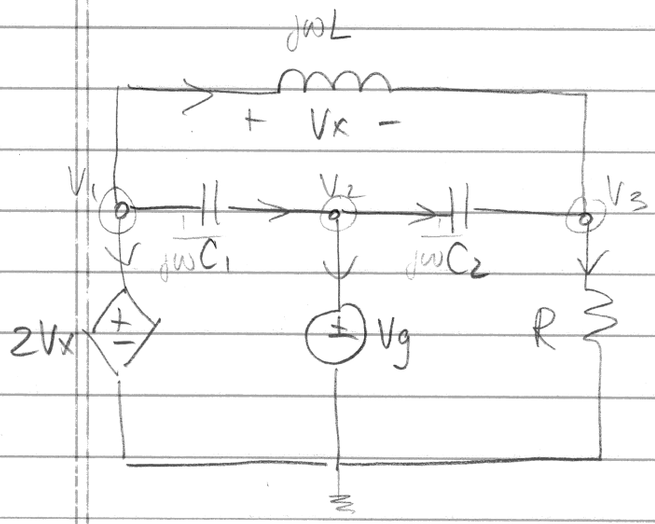
$\vec{V}_2 = 3.5 + j3.5 =$

$= 4.95 \angle 45°$

$v_2(t) = 2\cos t$

$v_2(t) = 4.95\cos(t + 45°)$

Example 2 (Thevenin)



$R = 1\,\Omega$

$C_1 = C_2 = 1\,F$

$L = 1\,H$

$V_g(t) = 5\cos t$; $\omega = 1\,rad/s$

$\vec{I}_R = \vec{V}_3 / R$

$\vec{I}_L = (\vec{V}_1 - \vec{V}_3)/j\omega L$

$\vec{I}_{C_1} = j\omega C_1 (\vec{V}_1 - \vec{V}_2)$

$\vec{I}_{C_2} = j\omega C_2 (\vec{V}_2 - \vec{V}_3)$

1, $\vec{I}_{Vx} = ?$

2, $\vec{I}_{Vg} = ?$

1) $\vec{I}_{Vx} + \vec{I}_L + \vec{I}_{C_1} = 0$

2) $-\vec{I}_{C_1} + \vec{I}_{Vg} + \vec{I}_{C_2} = 0$

3) $-\vec{I}_L - \vec{I}_{C_2} + \vec{I}_R = 0$

1) $\vec{V}_1 = 2\vec{V}_x = 2(\vec{V}_1 - \vec{V}_3)$

2) $\vec{V}_2 = \vec{V}_g$

3) $-\dfrac{(\vec{V}_1 - \vec{V}_3)}{j\omega L} - j\omega C(\vec{V}_2 - \vec{V}_3) + \dfrac{\vec{V}_3}{R} = 0$

1) $\vec{V}_1 - 2\vec{V}_3 = 0$

2) $\vec{V}_2 = \vec{V}_g = 5$

3) $-\dfrac{1}{j\omega L}\vec{V}_1 - j\omega C\vec{V}_2 + \vec{V}_3\left(\dfrac{1}{j\omega L} + j\omega C + \dfrac{1}{R}\right) = 0$

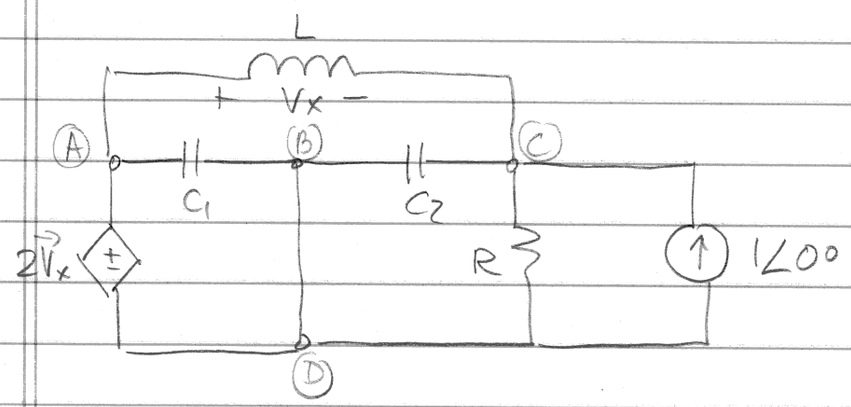
$$\vec{V}_1 - 2\vec{V}_3 = 0$$
$$\vec{V}_2 = 5$$
$$j\vec{V}_1 - j\vec{V}_2 + \vec{V}_3 = 0$$

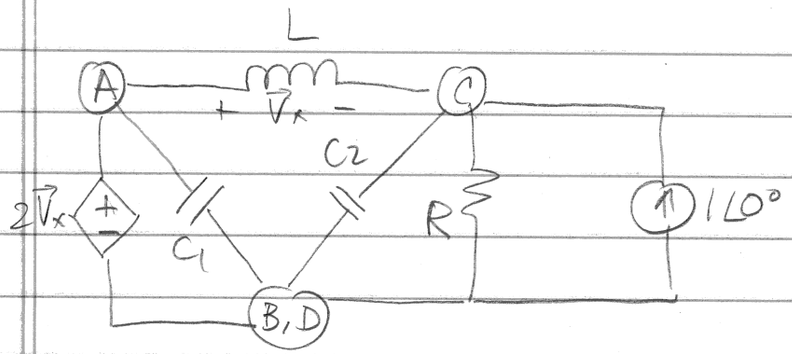
$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ j & -j & 1 \end{bmatrix} \begin{bmatrix} \vec{V}_1 \\ \vec{V}_2 \\ \vec{V}_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix}$$

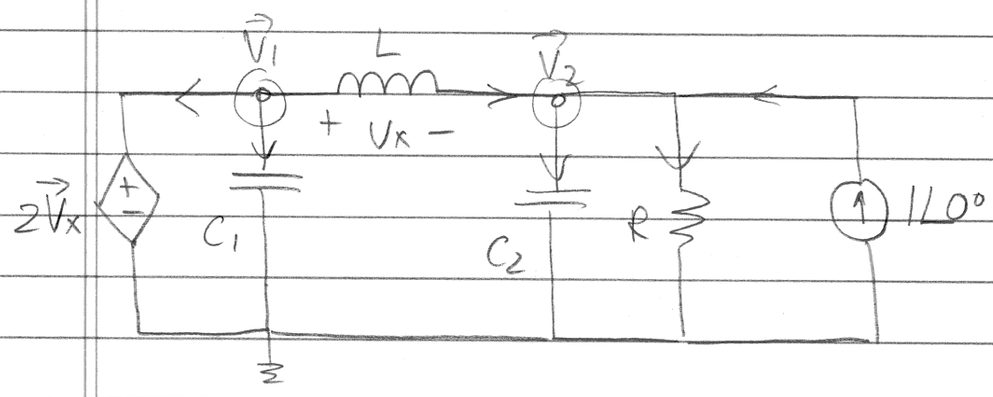
$$\vec{V}_1 = 4 + j2$$
$$\vec{V}_2 = 5$$
$$\vec{V}_3 = 2 + j$$

$$\Rightarrow \boxed{\vec{V}_{th} = \vec{V}_3 = 2 + j}$$

For Z_{th}:







$\vec{I}_R = \vec{V}_2/R$

$\vec{I}_L = (\vec{V}_1 - \vec{V}_2)/j\omega L$

$\vec{I}_{C_1} = j\omega C_1 \vec{V}_1$

$\vec{I}_{C_2} = j\omega C_2 \vec{V}_2$

$\vec{I}_{V_x} = ?$ I_1

1) $\vec{I}_{V_x} + \vec{I}_{C_1} + \vec{I}_L = 0$

2) $-\vec{I}_L + \vec{I}_{C_2} + \vec{I}_R - 1 = 0$

1) $\vec{V}_1 = 2\vec{V}_x = 2(\vec{V}_1 - \vec{V}_2)$

2) $-\frac{(\vec{V}_1 - \vec{V}_2)}{j\omega L} + j\omega C_2 \vec{V}_2 + \frac{\vec{V}_2}{R} = 1$

1) $\vec{V}_1 - 2\vec{V}_2 = 0$

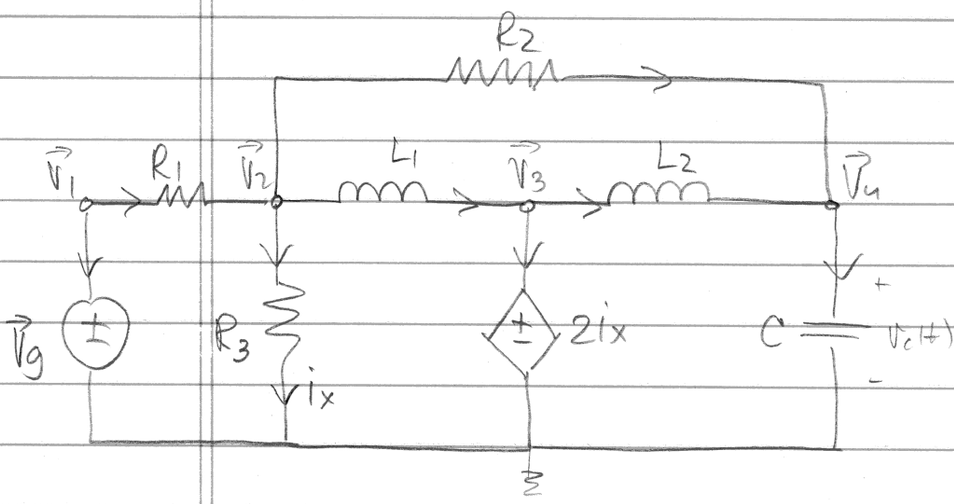
2) $-\frac{1}{j\omega L}\vec{V}_1 + \vec{V}_2\left(\frac{1}{j\omega L} + j\omega C_2 + \frac{1}{R}\right) = 1$

$\frac{1}{j} = -j$ $\quad \frac{1}{j} + j + 1$

$$\begin{bmatrix} 1 & -2 \\ j & 1 \end{bmatrix} \begin{bmatrix} \vec{V}_1 \\ \vec{V}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \rightarrow \begin{matrix} \vec{V}_1 = 0.4 - j0.8 \\ \vec{V}_2 = 0.2 - j0.4 \end{matrix}$$

$$Z_{th} = \frac{\vec{V}_2}{1\angle 0^\circ} = 0.2 - j0.4$$

Example 3



$R_1 = R_2 = R_3 = 1\,\Omega$

$L_1 = 1\,H$

$L_2 = 2\,H$

$C = \frac{1}{2}\,F$

$v_g(t) = 10\cos t$

Find $v_c(t)$

1) $\vec{I}_{Vg} + \vec{I}_{R_1} = 0$

2) $-\vec{I}_{R_1} + \vec{I}_{R_3} + \vec{I}_{R_2} + \vec{I}_{L_1} = 0$

3) $-\vec{I}_{L_1} + \vec{I}_{L_2} + \vec{I}_{2ix} = 0$

4) $-\vec{I}_{R_2} - \vec{I}_{L_2} + \vec{I}_C = 0$

$\vec{I}_{R_1} = (\vec{V}_1 - \vec{V}_2)/R_1$

$\vec{I}_{R_2} = (\vec{V}_2 - \vec{V}_4)/R_2$

$\vec{I}_{R_3} = \vec{V}_2/R_3$

$\vec{I}_{L_1} = (\vec{V}_2 - \vec{V}_3)/j\omega L_1$

$\vec{I}_{L_2} = (\vec{V}_3 - \vec{V}_4)/j\omega L_2$

$\vec{I}_C = j\omega C \vec{V}_4$

1. $\vec{I}_{Vg} = ?$
2. $\vec{I}_{2ix} = ?$

1) $\vec{V}_1 = \vec{V}_g = 10$

2) $-\frac{(\vec{V}_1 - \vec{V}_2)}{R_1} + \frac{\vec{V}_2}{R_3} + \frac{(\vec{V}_2 - \vec{V}_4)}{R_2} + \frac{(\vec{V}_2 - \vec{V}_3)}{j\omega L_1} = 0$

3) $\vec{V}_3 = 2\vec{I}_x = 2\vec{I}_{R_3} = \frac{2\vec{V}_2}{R_3}$

4) $-\frac{(\vec{V}_2 - \vec{V}_4)}{R_2} - \frac{(\vec{V}_3 - \vec{V}_4)}{j\omega L_2} + j\omega C \vec{V}_4 = 0$

1) $\vec{V}_1 = 10$

2) $-\frac{1}{R_1}\vec{V}_1 + \vec{V}_2\left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_2} + \frac{1}{j\omega L_1}\right) - \frac{1}{j\omega L_1}\vec{V}_3 - \frac{1}{R_2}\vec{V}_4 = 0$

$1 \qquad 3 + \frac{1}{j} = 3 - j \qquad = -j \qquad 1$

3) $\frac{2}{R_3}\vec{V}_2 - \vec{V}_3 = 0$

$=2$

4) $-\frac{1}{R_2}\vec{V}_2 - \frac{1}{j\omega L_2}\vec{V}_3 + \vec{V}_4\left(\frac{1}{R_2} + \frac{1}{j\omega L_2} + j\omega C\right) = 0$

$1 \qquad \frac{1}{j^2} = -j/2 \qquad 1 - j/2 + j/2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 3-j & j & -1 \\ 0 & 2 & -1 & 0 \\ 0 & -1 & 0.5j & 1 \end{bmatrix} \begin{bmatrix} \vec{V}_1 \\ \vec{V}_2 \\ \vec{V}_3 \\ \vec{V}_4 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{array}{l} \vec{V}_1 = 10 \\ \vec{V}_2 = 2.5 - j2.5 \\ \vec{V}_3 = 5 - j5 \\ \vec{V}_4 = -j5 \end{array}$$

$\vec{V}_4 = -j5 = 5\angle -90^\circ \Rightarrow v_c(t) = 5\cos(t - 90^\circ) = 5\sin t$