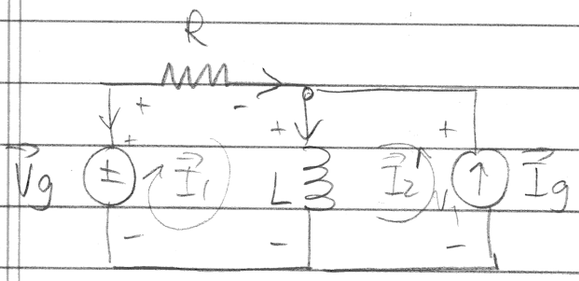
The Loop Method

Example 0



$R = 1\Omega$; $L = 1H$

$v_g = 2\cos t \rightarrow \vec{V}_g = 2$

$i_g = 5\cos t \rightarrow \vec{I}_g = 5$

$$-\vec{V}_g + \vec{V}_R + \vec{V}_L = 0$$

$$\vec{V}_R = R\vec{I}_R = R\vec{I}_1$$

$$\vec{V}_L = j\omega L\vec{I}_L = j\omega L(\vec{I}_1 + \vec{I}_2)$$

$$-\vec{V}_{Ig} + \vec{V}_L = 0$$

$\vec{V}_{Ig} = ?$

$$-\vec{V}_g + R\vec{I}_1 + j\omega L(\vec{I}_1 + \vec{I}_2) = 0$$

R - exclusive to loop 1, + terminal

L - shared by L_1 & L_2, + -

$$\vec{I}_2 = \vec{I}_g = 5$$

$$\vec{I}_1(\underbrace{1}_{R} + \underbrace{j\omega L}_{j}) + j\omega L\vec{I}_2 = 2$$

$$\vec{I}_2 = 5$$

$$\begin{bmatrix} 1+j & +j \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \vec{I}_1 \\ \vec{I}_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

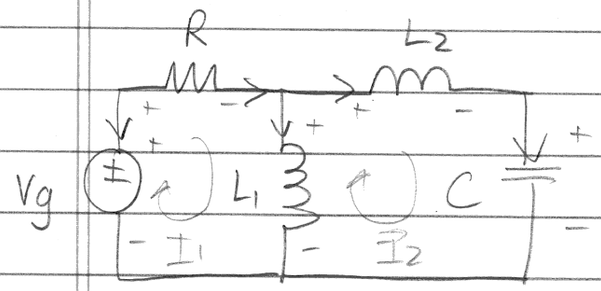
$$\vec{I}_1 = -1.5 - j3.5$$

$$\vec{I}_2 = 5$$

$$\vec{I}_1 = 3.8079\angle -113.2^\circ \rightarrow i_1(t) = 3.808\cos(t - 113.2^\circ)$$

$$i_2(t) = 5\cos t$$

Example 1



$R = 1\,\Omega$

$L_1 = L_2 = 1\,H$

$C = 2\,F$

$v_g(t) = 5\cos t$

$$-\vec{V}_g + \vec{V}_R + \vec{V}_{L_1} = 0$$

$$\vec{V}_{L_2} + \vec{V}_C - \vec{V}_{L_1} = 0$$

$$\vec{V}_R = R\vec{I}_R = R\vec{I}_1$$

$$\vec{V}_{L_1} = j\omega L_1 \vec{I}_{L_1} = j\omega L_1(\vec{I}_1 - \vec{I}_2)$$

$$\vec{V}_{L_2} = j\omega L_2 \vec{I}_{L_2} = j\omega L_2 \vec{I}_2$$

$$\vec{V}_C = \frac{1}{j\omega C}\vec{I}_C = \frac{1}{j\omega C}\vec{I}_2$$

$$-\vec{V}_g + R\vec{I}_1 + j\omega L_1(\vec{I}_1 - \vec{I}_2) = 0$$

$$j\omega L_2 \vec{I}_2 + \frac{1}{j\omega C}\vec{I}_2 - j\omega L_1(\vec{I}_1 - \vec{I}_2) = 0$$

$$\Rightarrow \quad \underbrace{(1+j)}\,\vec{I}_1 - \underbrace{j}\,\vec{I}_2 = \vec{V}_g$$

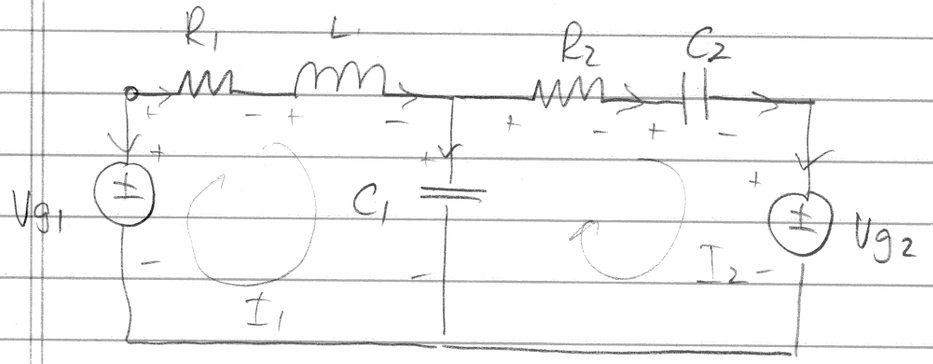
$$\underbrace{-j}\,\vec{I}_1 + \underbrace{(1.5j)}\,\vec{I}_2 = 0$$

$$\begin{bmatrix} 1+j & -j \\ -j & 1.5j \end{bmatrix}\begin{bmatrix} \vec{I}_1 \\ \vec{I}_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix} \rightarrow \begin{matrix} \vec{I}_1 = 4.5 - j1.5 \\ \vec{I}_2 = 3 - j \end{matrix}$$

$$\vec{I}_1 = 4.74\angle -18.4^\circ \rightarrow i_1(t) = 4.74\cos(t - 18.4^\circ)$$

$$\vec{I}_2 = 3.16\angle -18.4^\circ \qquad i_2(t) = 3.16\cos(t - 18.4^\circ)$$

Example 2



$v_{g1} = 5\cos 2t$

$v_{g2} = \cos 2t$

$R_1 = R_2 = 1\Omega$

$L = 1H$

$C_1 = C_2 = 1F$

$$-\vec{V}_{g1} + \vec{V}_{R1} + \vec{V}_L + \vec{V}_{C1} = 0$$

$$\vec{V}_{R2} + \vec{V}_{C2} + \vec{V}_{g2} - \vec{V}_{C1} = 0$$

$$\frac{1}{j} = -j \quad \Rightarrow$$

$$\vec{V}_{R1} = R_1 \vec{I}_{R1} = R_1 \vec{I}_1$$

$$\vec{V}_{R2} = R_2 \vec{I}_{R2} = R_2 \vec{I}_2$$

$$\vec{V}_L = j\omega L \vec{I}_L = j\omega L \vec{I}_1$$

$$\vec{V}_{C1} = \frac{1}{j\omega C_1} \vec{I}_{C1} = \frac{1}{j\omega C_1}(\vec{I}_1 - \vec{I}_2)$$

$$\vec{V}_{C2} = \frac{1}{j\omega C_2} \vec{I}_{C2} = \frac{1}{j\omega C_2} \vec{I}_2$$

$$-\vec{V}_{g1} + R_1\vec{I}_1 + j\omega L\vec{I}_1 + \frac{1}{j\omega C_1}(\vec{I}_1 - \vec{I}_2) = 0$$

$$R_2\vec{I}_2 + \frac{1}{j\omega C_2}\vec{I}_2 + \vec{V}_{g2} - \frac{1}{j\omega C_1}(\vec{I}_1 - \vec{I}_2) = 0$$

$$\vec{I}_1\left(R_1 + j\omega L + \frac{1}{j\omega C_1}\right) - \frac{1}{j\omega C_1}\vec{I}_2 = \vec{V}_{g1} = 5$$

$$-\frac{1}{j\omega C_1}\vec{I}_1 + \vec{I}_2\left(R_2 + \frac{1}{j\omega C_2} + \frac{1}{j\omega C_1}\right) = -\vec{V}_{g2} = -1$$

$$\begin{bmatrix} 1+j1.5 & j0.5 \\ j0.5 & 1-j \end{bmatrix} \begin{bmatrix} \vec{I}_1 \\ \vec{I}_2 \end{bmatrix} = \begin{bmatrix} 5 \\ -1 \end{bmatrix}$$

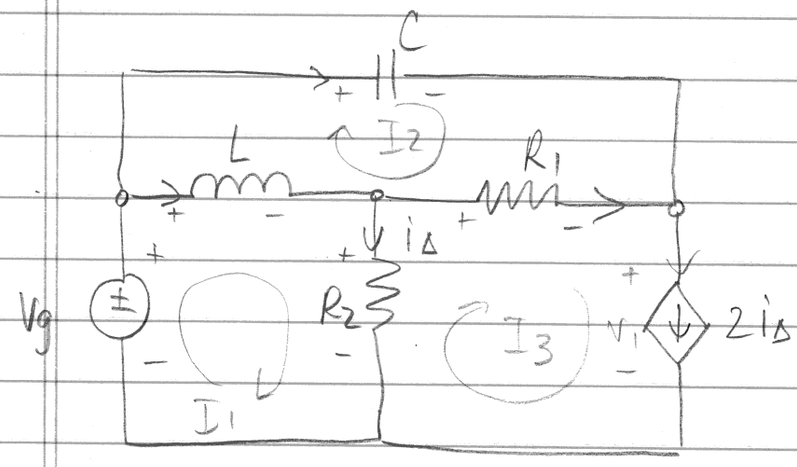
$$\vec{I}_1 = 1.472 - j1.904$$

$$\vec{I}_2 = -0.608 - j1.344$$

$$\vec{I}_1 = 2.4067 \angle -52.3^\circ \rightarrow i_1(t) = 2.4067\cos(2t - 52.3^\circ)$$

$$\vec{I}_2 = 1.4751 \angle -114.34^\circ \rightarrow i_2(t) = 1.4751\cos(2t - 114.34^\circ)$$

Example 3



$R_1 = R_2 = 1\Omega$

$L = 0.5\,H$

$C = 1\,F$

$\omega = 2\,rad/s$

$v_g(t) = 5\sin 2t = 5\cos(2t - 90°)$

$\vec{V}_{R1} = R_1 \vec{I}_{R1} = R_1(\vec{I}_3 - \vec{I}_2)$

$\vec{V}_{R2} = R_2 \vec{I}_{R2} = R_2(\vec{I}_1 - \vec{I}_3)$

$\vec{V}_L = j\omega L \vec{I}_L = j\omega L(\vec{I}_1 - \vec{I}_2)$

$\vec{V}_C = \frac{1}{j\omega C}\vec{I}_2$

V_Δ $\vec{V}_{2i_\Delta} = ?$

1) $-\vec{V}_g + \vec{V}_L + \vec{V}_{R2} = 0$

2) $\vec{V}_C - \vec{V}_{R1} - \vec{V}_L = 0$

3) $\vec{V}_{R1} + \vec{V}_{2i_\Delta} - \vec{V}_{R2} = 0$

1) $-\vec{V}_g + j\omega L(\vec{I}_1 - \vec{I}_2) + R_2(\vec{I}_1 - \vec{I}_3) = 0$

2) $\frac{1}{j\omega C}\vec{I}_2 - R_1(\vec{I}_3 - \vec{I}_2) - j\omega L(\vec{I}_1 - \vec{I}_2) = 0$

3) $\vec{I}_3 = 2\vec{I}_\Delta = 2(\vec{I}_1 - \vec{I}_3)$

1) $\vec{I}_1(R_2 + j\omega L) - j\omega L\vec{I}_2 - R_2\vec{I}_3 = \vec{V}_g = -j5$

2) $-j\omega L\vec{I}_1 + \vec{I}_2(R_1 + \frac{1}{j\omega C} + j\omega L) - R_1\vec{I}_3 = 0$

3) $2\vec{I}_1 - 3\vec{I}_3 = 0$

$$\begin{bmatrix} (1+j) & -j & -1 \\ -j & 1+0.5j & -1 \\ 2 & 0 & -3 \end{bmatrix} \begin{bmatrix} \vec{I}_1 \\ \vec{I}_2 \\ \vec{I}_3 \end{bmatrix} = \begin{bmatrix} -j5 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{aligned} \vec{I}_1 &= -0.4412 - j5.7353 \\ \vec{I}_2 &= 2.6471 - j5.5882 \\ \vec{I}_3 &= -0.2941 - j3.8235 \end{aligned}$$

$\vec{I}_1 = 5.7522 \angle -94.39^\circ \rightarrow i_1(t) = 5.752 \cos(2t - 94.39^\circ)$

$\vec{I}_2 = 6.1835 \angle -64.65^\circ \rightarrow i_2(t) = 6.1835 \cos(2t - 64.65^\circ)$

$\vec{I}_3 = 3.8348 \angle -94.39^\circ \quad i_3(t) = 3.8348 \cos(2t - 94.39^\circ)$